COINTEGRATED OSCILLATING SYSTEMS

Østergaard, Jacob (speaker/presenter)
University of Copenhagen, Denmark, ostergaard@math.ku.dk
Ditlevsen, Susanne *University of Copenhagen, Denmark*
Rahbek, Anders *University of Copenhagen, Denmark*

Cointegration; Oscillating systems; Phase process; Synchronization:

Oscillating systems has attracted significant attention from researchers for a long time. Single cell systems have been the center of attention for more than a century, however interaction within cell clusters have progressively gained consideration as well. Presently, no robust statistical framework for modelling clusters of synchronizing cells exist, and synchronization is not well understood. Since the discovery by Huygens in the 17th century, this phenomenon is still a puzzle for researchers. The importance of understanding synchronization mechanisms can be seen by the ubiquitous presence in various natural phenomenons [3], from EEG signals, to swarms of fireflies, and even concert audiences.

Cointegration has been applied in econometrics since the introduction in 1981 by Nobel Laureate Clive Granger. The following decades led to extensive research in the field, and today the *Johansen test* has become a standard statistical procedure for inference on multivariate nonstationary and cointegrated time series. Cointegration theory has since been developed further to include switching regime models, continuous time models, nonlinear cointegration, but until now applications has mainly been secluded to the field of econometrics. Some attempts at bridging cointegration with dynamical systems have been tried [1], [2], but neither has led to a fully interpretable result.

We show how cointegration can be applied to analyze a multivariate system of coupled oscillators, specified as a general system of oscillating processes, exhibiting interaction in the phase processes. We define the system of coupled phase processes through a p-dimensional continuous time stochastic differential equation

$$d\phi_t = \big(f(\phi_t) + \mu\big)dt + \Sigma dW_t,$$

where the coupling of phases is imposed through the function f. This formulation covers among others the celebrated Kuramoto model, as a special case where f is nonlinear.

In our study we specify a linear interaction in the phases and present a simulation study on an assumed observed data generating process $z_t = \big(\gamma_t \cos(\phi_t), \gamma_t \sin(\phi_t)\big)'$, where γ_t is a corresponding amplitude process. We perform a cointegration analysis of the latent ϕ_t process obtained from a numerical solution of z_t, and demonstrate how we can identify the interaction in the system, and statistically conclude on the dependency of the oscillators.

Finally we discuss our current research on applications of cointegration to systems with non-linear coupling functions.

References

[1] Dahlhaus, R., Neddermeyer, J. (2012) *On The Relationship Between The Theory Of Cointegration And The Theory Of Phase Synchronization*, Working paper.

[2] Kammerdiner, A.R., Pardalos, P.M. (2012) *Analysis of multichannel EEG recordings based on generalized phase synchronization and cointegrated VAR*, Computational Neuroscience.

[3] Pikovsky, A., Rosenblum, M., Kurths, J. (2001) Synchronization: A Universal Concept in Nonlinear Sciences, Cambridge.