NONPARAMETRIC BAYESIAN POSTERIOR CONTRACTION RATES FOR DISCRETELY OBSERVED SCALAR DIFFUSIONS

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Diffusion Processes; Nonparametric Bayesian Statistics; Low-Frequency Observations; Contraction Rates:

We consider nonparametric Bayesian inference in a reflected diffusion model $\mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t$, with discretely sampled observations $X_0, X_\Delta, \dots, X_{n\Delta}$. We analyse the nonlinear inverse problem corresponding to the 'low-frequency sampling' regime where $\Delta > 0$ is fixed and $n \to \infty$. A general theorem is proved that gives conditions for prior distributions Π on the diffusion coefficient σ and the drift function b that ensure minimax optimal contraction rates of the posterior distribution over Hölder-Sobolev smoothness classes. These conditions are verified for natural examples of nonparametric prior distributions, including uniform wavelet series and Gaussian priors. It is argued that hierarchical Bayesian models that simultaneously correctly model the invariant measure are preferable. For the proofs we derive new concentration inequalities for empirical processes arising from discretely observed diffusions that are of independent interest.

References

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