ON MULTIPLE CHANGE-POINT ESTIMATION FOR POISSON PROCESS

Top, Alioune
Université du Maine/France, Alioune.Top.Etu@univ-lemans.fr

Inhomogeneous Poisson process; change-point; Bayesian estimator; maximum likelihood estimator; likelihood ratio process:

We consider the model of n independent observation of an inhomogeneous Poisson processes $X^{(n)} = (X_1, \ldots, X_n)$ where $X_j = \{X_j(t),\, 0 \leq t \leq T\}$, $j = 1, \ldots, n$ are Poisson processes. We suppose that the intensity function of the observed Poisson processes is the same

$$\lambda(\theta, t) = \lambda_0 + \lambda_1(t) 1_{\{\theta \leq t \leq \theta + \tau_0\}} \tag{1}$$

The parameter $\theta \in \Theta = (\alpha, \beta), 0 \leq \alpha \leq \beta \leq \beta + \tau_0 < \tau = T - \tau_0$ is unknown and corresponds to the location of a jump in the intensity function $\lambda(\theta, t)$ on $[0, \tau]$. We suppose that λ_0 is a positive constant and the function $\lambda_1(\cdot)$ is increasing, strictly positive and continuous for all $t \in [0, \tau]$. So the model (1) depends on θ and has two jumps and we are interested by the asymptotic properties of the MLE $\hat{\theta}_n$ and Bayesian estimator (BE) $\tilde{\theta}_n$ of θ as $n \longrightarrow +\infty$.

The centerpiece of the method is the weak convergence of the normalized likelihood ratio process to an exponential functional of a two-sided difference of two Poisson processes driven by some parameters in a suitable metric space. In particular, we check the convergence of finite-dimensional distributions and the tightness of the corresponding family measures in the Skorohod space $\mathbf{D}_0(\mathbf{R})$. This result enables us to prove that ours estimators of the jumps location converge with exact rate n^{-1} (better than in the regular case), i.e. Uniformly on $\theta \in \mathbf{K}$ ($\mathbf{K}$ an arbitrary compact in Θ) we obtain:

the consistency

$$\mathbf{P}_{\theta_0} - \lim_{n \to +\infty} \tilde{\theta}_n = \theta, \qquad \mathbf{P}_{\theta_0} - \lim_{n \to +\infty} \hat{\theta}_n = \theta;$$

the convergence in Law

$$\mathcal{L}_\theta\left\{n\left(\tilde{\theta}_n - \theta_0\right)\right\} \Rightarrow \mathcal{L}(\tilde{u}_\rho) \qquad \mathcal{L}_\theta\left\{n\left(\hat{\theta}_n - \theta_0\right)\right\} \Rightarrow \mathcal{L}(\hat{u}_\rho).$$

where the variables $\tilde{u}_\rho$, $\hat{u}_\rho$ satisfy

$$\tilde{u}_\rho = \frac{\int_{-\infty}^{+\infty} v Z_\rho^*(v)\, dv}{\int_{-\infty}^{+\infty} Z_\rho^*(v)\, dv}, \qquad \max\Big(Z_\rho^*(\hat{u}_\rho -), Z_\rho^*(\hat{u}_\rho +)\Big) = \sup_{v \in \mathbf{R}} Z_\rho^*(v),$$

and $Z_\rho^*(v)$ is a limit process.

It is shown that the both estimators have convergence of moments and the BE are asymptotically efficient. Finally we present Monte-Carlo simulations to illustrate the obtained results and show that Bayesian estimator outperforms the MLE. Its concur also the i.i.d. case with one point of singularity (see [1] and [2])where it was mentioned that the Bayesian estimators are generally more efficient that the MLE estimators in Change-Point type estimation problems.

References

[1] Ibragimov, I. A., and Khasminskii, R. Z.,(1981) *Statistical Estimation. Asymptotic Theory*, Springer, New York.

[2] Kutoyants, Yu. A.,(1998) *Statistical Inference for Spatial Poisson Processes*, Lecture Notes in Statistics **134**, Springer-Verlag, New York.