

PARAMETRIC INFERENCE FOR DISCRETE OBSERVATIONS OF DIFFUSIONS WITH RANDOM EFFECTS IN THE DRIFT OR IN THE DIFFUSION COEFFICIENT

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Maximum likelihood estimator; Mixed-effects models; Stochastic differential equations

Abstract

We consider N independent stochastic processes $(X_i(t), t \in [0, T])$, $i = 1, \dots, N$, defined by a stochastic differential equation with drift term depending on a random vector Φ_i and diffusion coefficient depending on another random effect Ψ_i :

$$dX_i(t) = \Phi_i' b(X_i(t)) dt + \Psi_i \sigma(X_i(t)) dW_i(t), \quad X_i(0) = x, \quad i = 1, \dots, N,$$

where (W_1, \dots, W_N) are N independent Wiener processes, $((\Phi_i, \Psi_i), i = 1, \dots, N)$ are N i.i.d. $\mathbb{R}^d \times (0, +\infty)$ -valued random variables, $((\Phi_i, \Psi_i), i = 1, \dots, N)$ and (W_1, \dots, W_N) are independent and x is a known real value. The functions $\sigma(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ and $b(\cdot) = (b_1(\cdot), \dots, b_d(\cdot))' : \mathbb{R} \rightarrow \mathbb{R}^d$ are known. Each process $(X_i(t))$ represents an individual and the $d + 1$ -dimensional random vector (Φ_i, Ψ_i) represents the random effect of individual i and is discretely observed on a fixed-length time interval $[0, T]$ with $T > 0$ at n times $t_j = jT/n$. We consider parametric distributions for the random effects and estimate the unknown parameters from the observations $\{X_i(t_j), j = 1, \dots, n, i = 1, \dots, N\}$, as n, N go to infinity. We study cases that give rise to explicit approximations of the likelihood functions:

- (1) Random effect in the drift: $\Phi_i \sim \mathcal{N}_d(\mu, \gamma^{-1}\Omega)$ and fixed effect in the diffusion coefficient $\Psi_i = \psi = \gamma^{-1/2}$.
- (2) Fixed effect in the drift $\Phi_i = \varphi$ and random effect in the diffusion coefficient $\Psi_i = \Gamma_i^{-1/2}$ and that Γ_i has Gamma distribution $G(a, \lambda)$.

In each case, we characterize the asymptotic behaviour of the associated estimators.

References

- [1] Delattre M., Genon-Catalot V., Larédo C. (2016) *Parametric inference for discrete observations of diffusions with random effects in the drift or in the diffusion coefficient*,
- [2] Delattre M., Genon-Catalot V., Larédo C. (2016) *Parametric inference for discrete observations of diffusions with random effects in the drift and in the diffusion coefficient*.