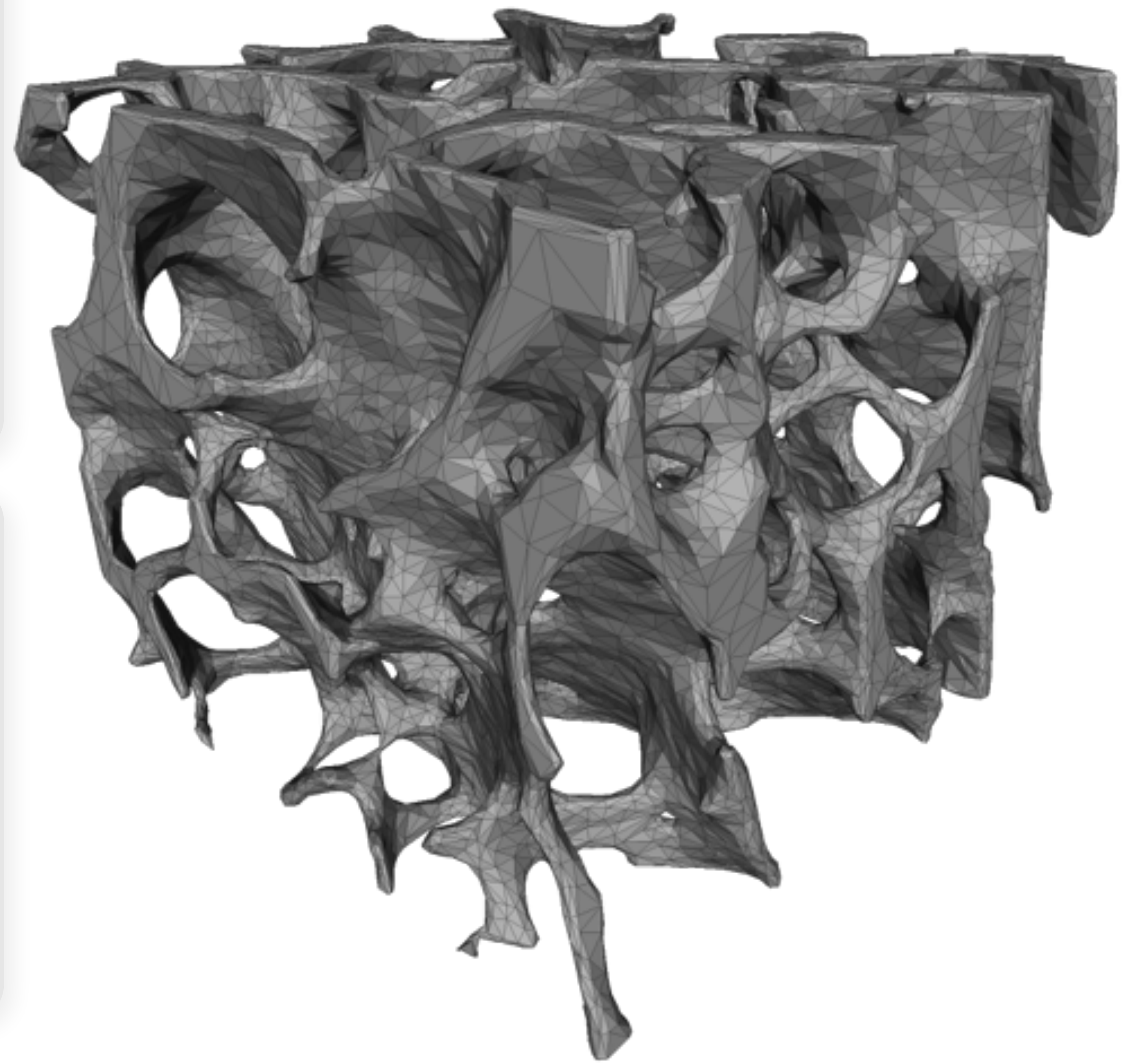
Modelling Trabecular Bone

$$
\begin{aligned}
-\nabla \cdot \boldsymbol{\sigma}(\mathbf{u}) &= 0 \quad \text{in } \Omega, \\
\boldsymbol{\sigma}(\mathbf{u})\mathbf{n} &= 0 \quad \text{on } \partial\Omega_{\text{int}}, \\
\mathbf{u} \cdot \mathbf{n} &= g_n \quad \text{on } \partial\Omega_{\text{box}}, \\
\boldsymbol{\sigma}(\mathbf{u})\mathbf{n} \cdot \mathbf{t} &= 0 \quad \text{on } \partial\Omega_{\text{box}}.
\end{aligned}
$$

$$J(\mathbf{u}) = \frac{1}{E} \frac{1}{g_n^{\text{top}}} \frac{h_{\text{box}}}{|\Omega_{\text{box}}|} \int_\Omega \boldsymbol{\sigma}_{33} d\mathbf{x},$$

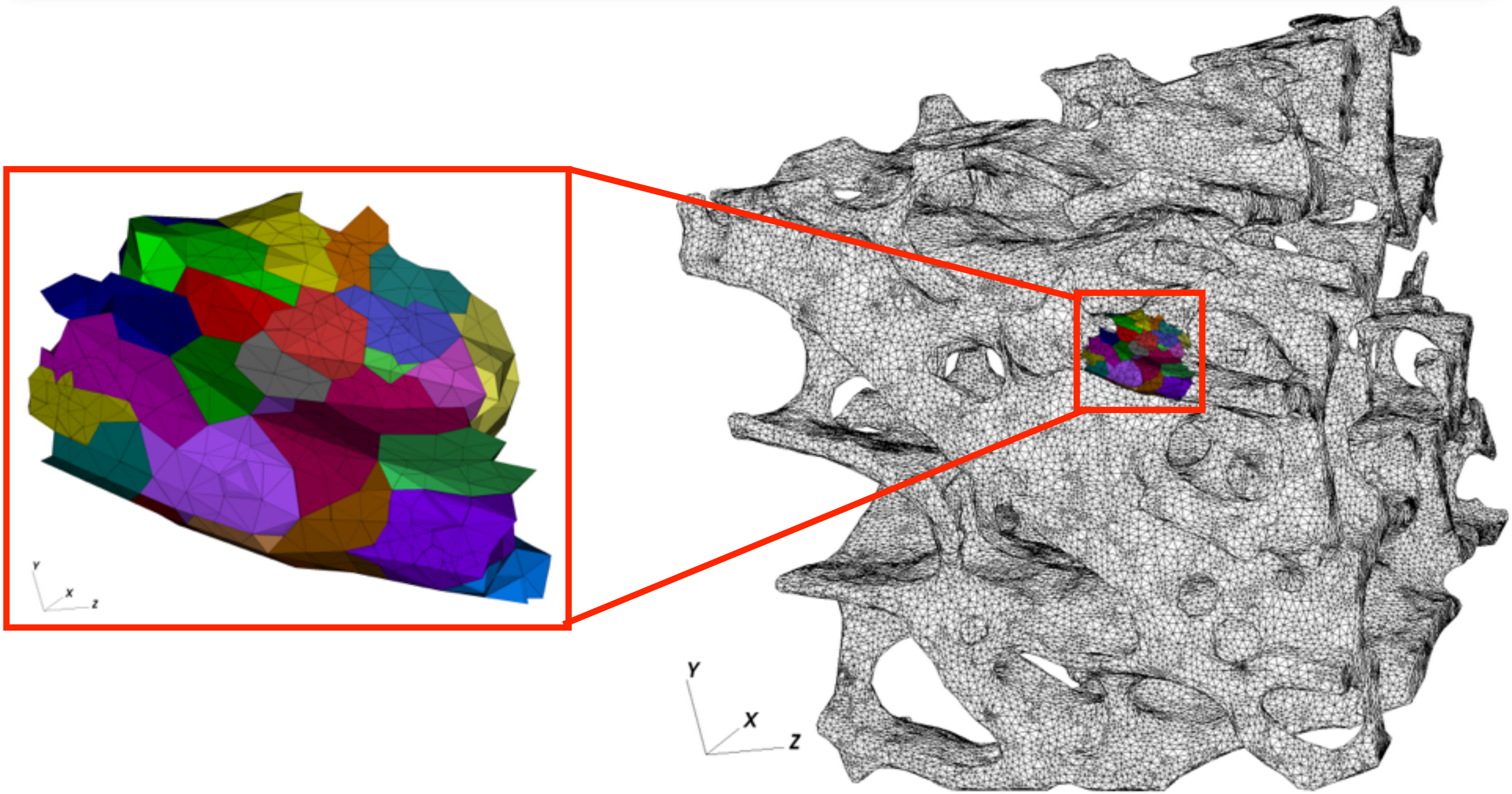
$$g_n^{\text{top}} = 0.01 h_{\text{box}}.$$

$$E = 10\text{GPa and } \nu = 0.3.$$



Verhoosel, van Zwieten, van Rietbergen & de Borst 2015, Collis & H 2016

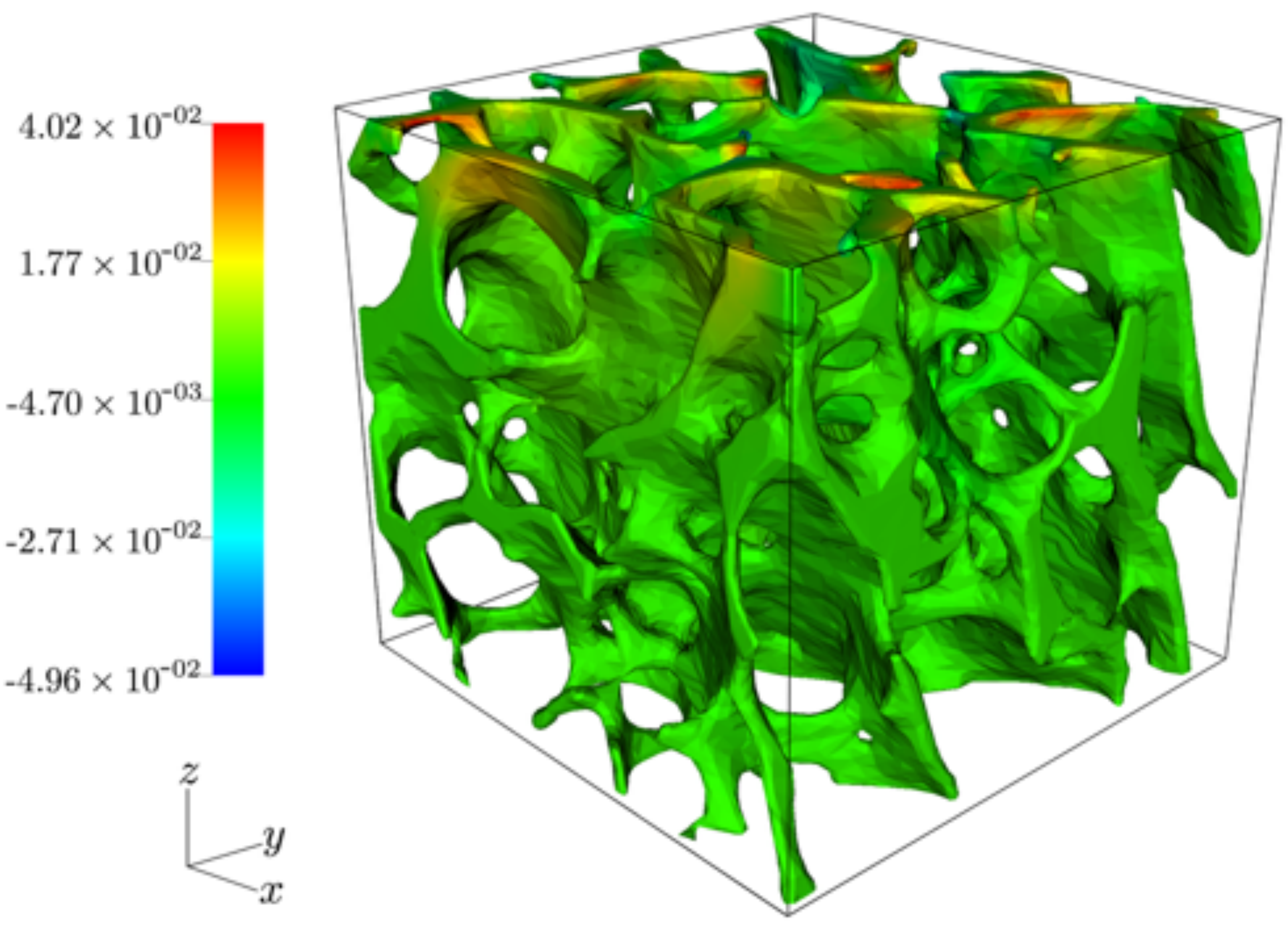
Fine mesh consists of 1.2M elements; Agglomerated mesh with 8K elements.



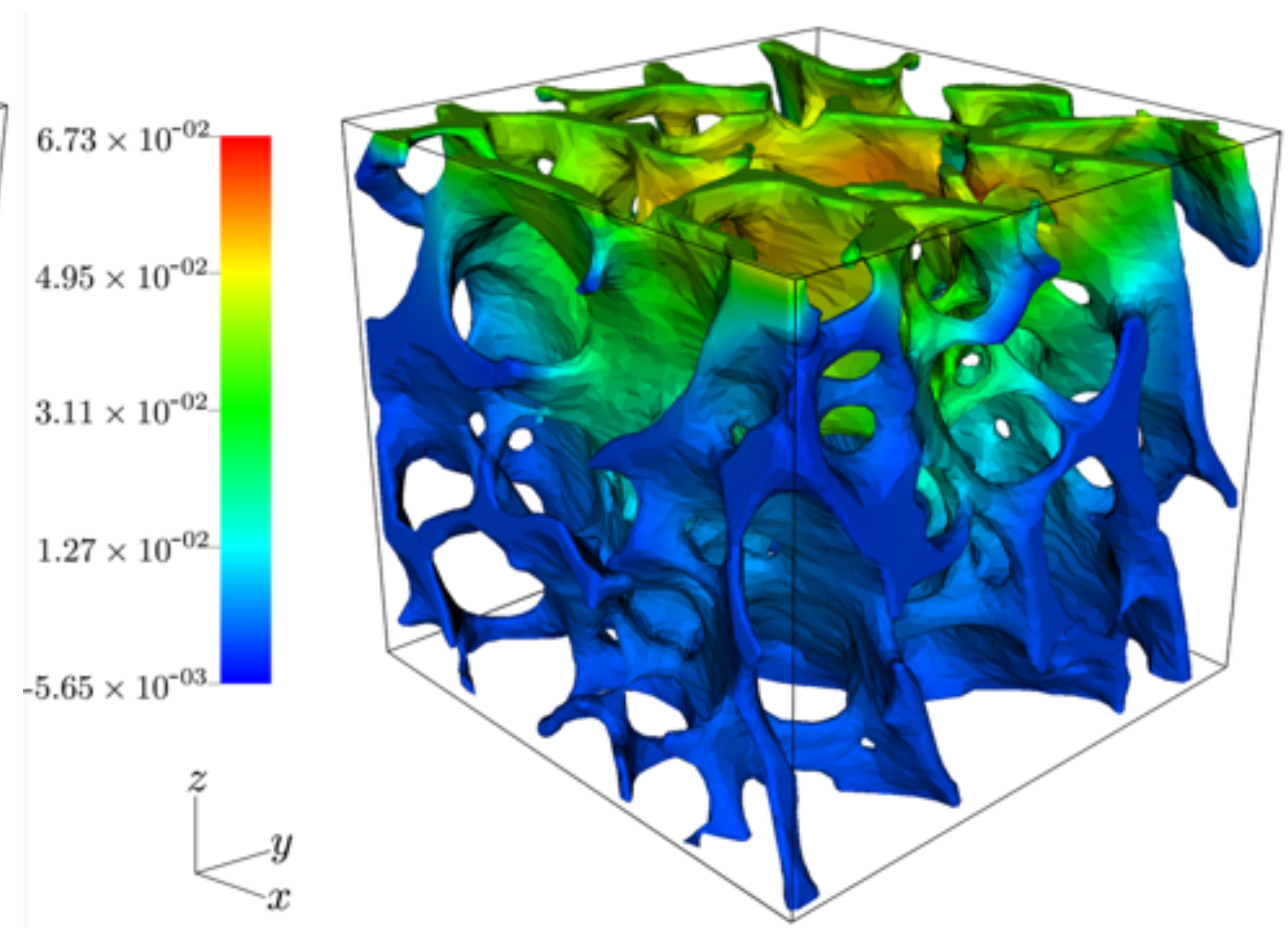
Verhoosel, van Zwieten, van Rietbergen & de Borst 2015, Collis & H 2016

Modelling Trabecular Bone

Fine mesh consists of 1.2M elements; Agglomerated mesh with 8K elements.



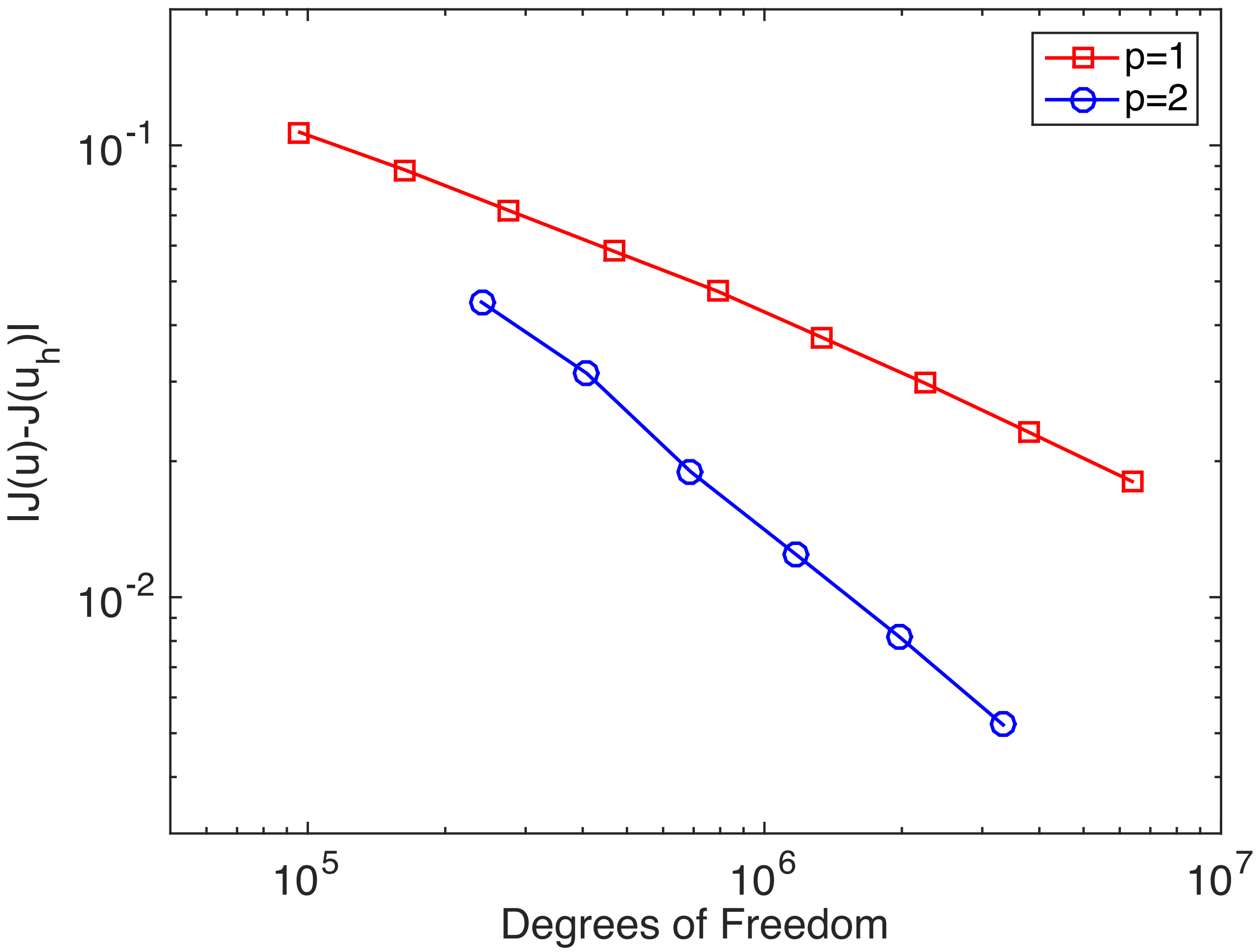
u_1

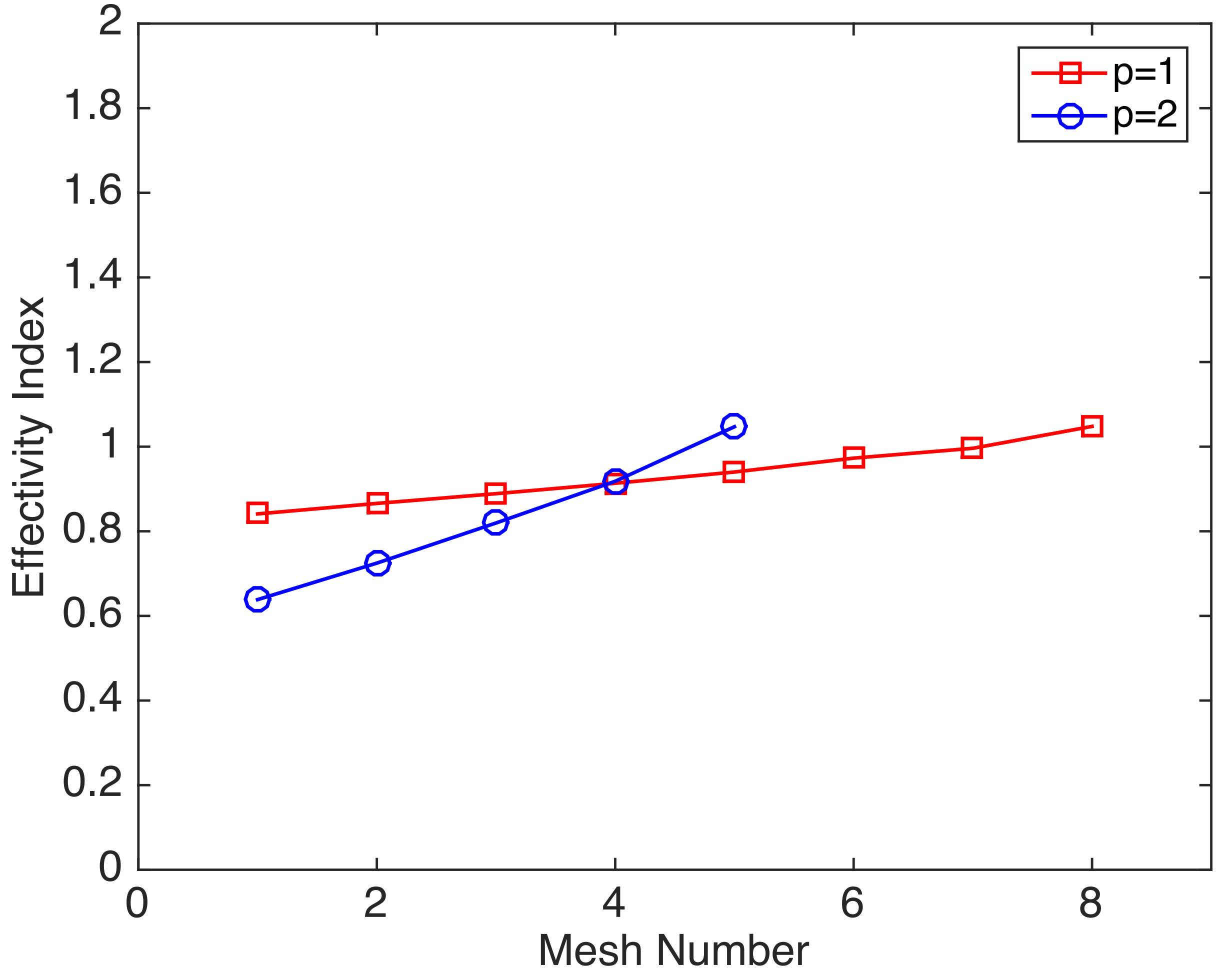


u_3

Verhoosel, van Zwieten, van Rietbergen & de Borst 2015, Collis & H 2016

Modelling Trabecular Bone

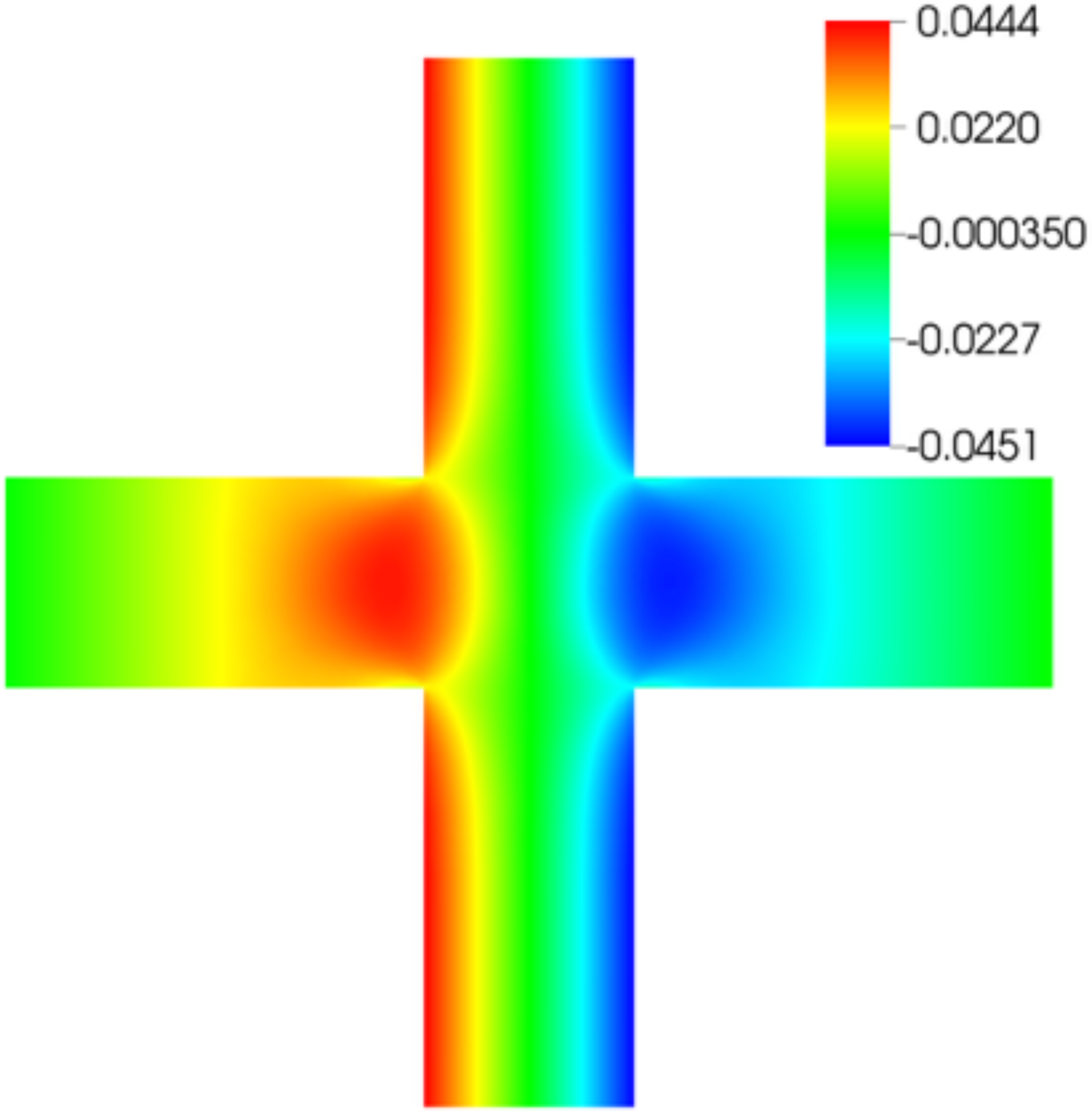




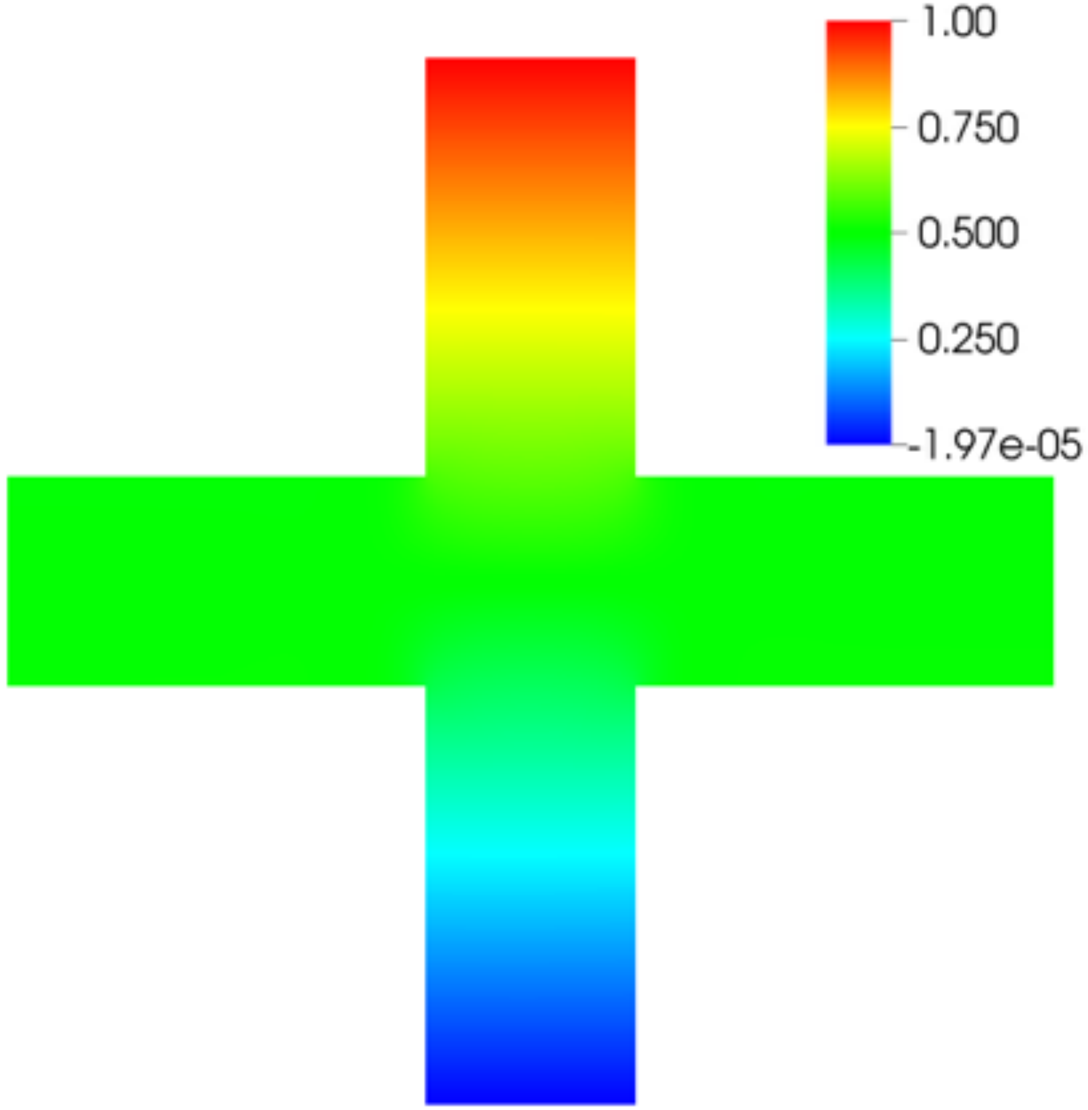
Verhoosel, van Zwieten, van Rietbergen & de Borst 2015, Collis & H 2016

Cross Domain in 2D

Fine mesh consists of 111K elements; Agglomerated mesh with 32 elements.

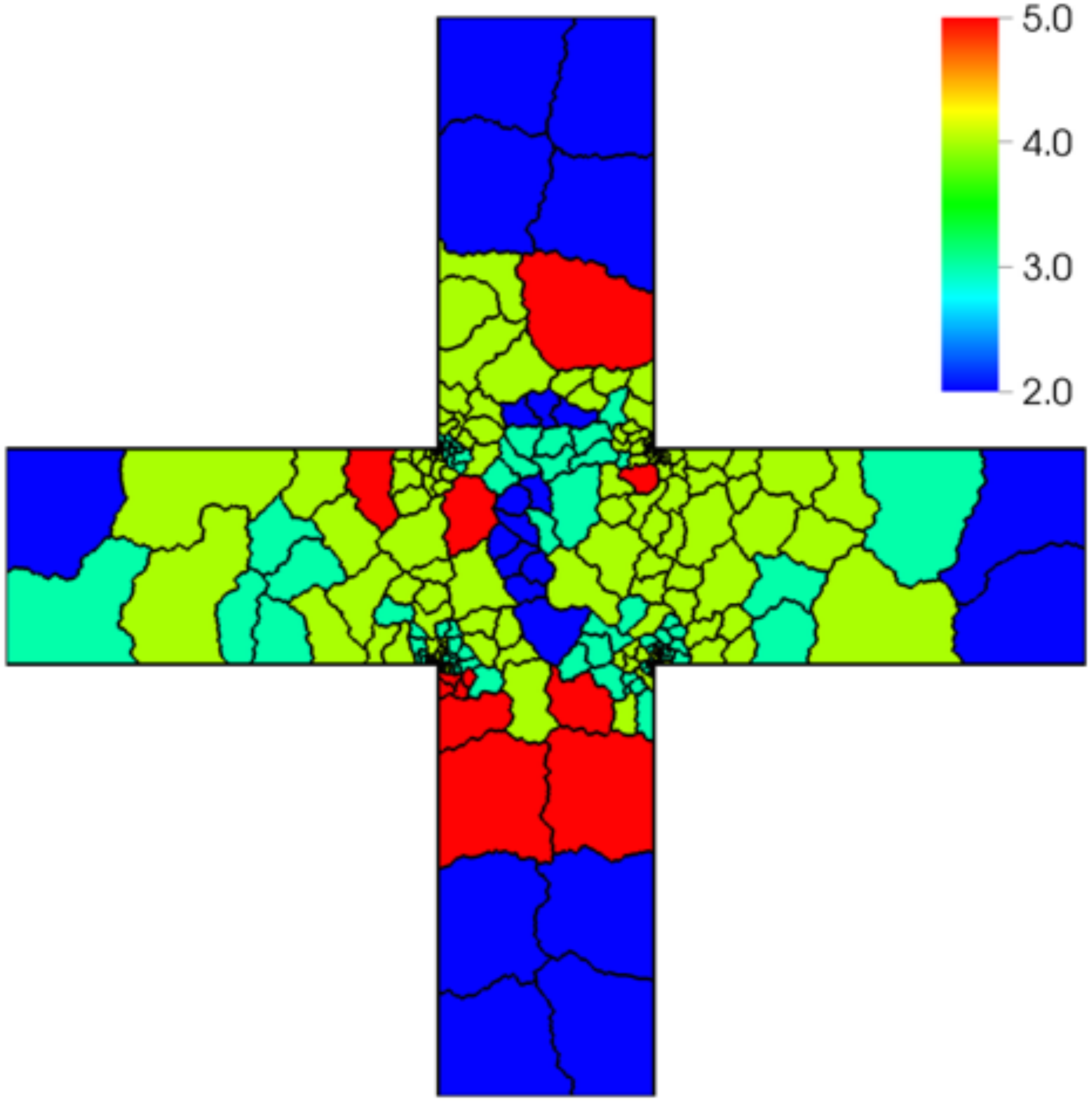


u_1

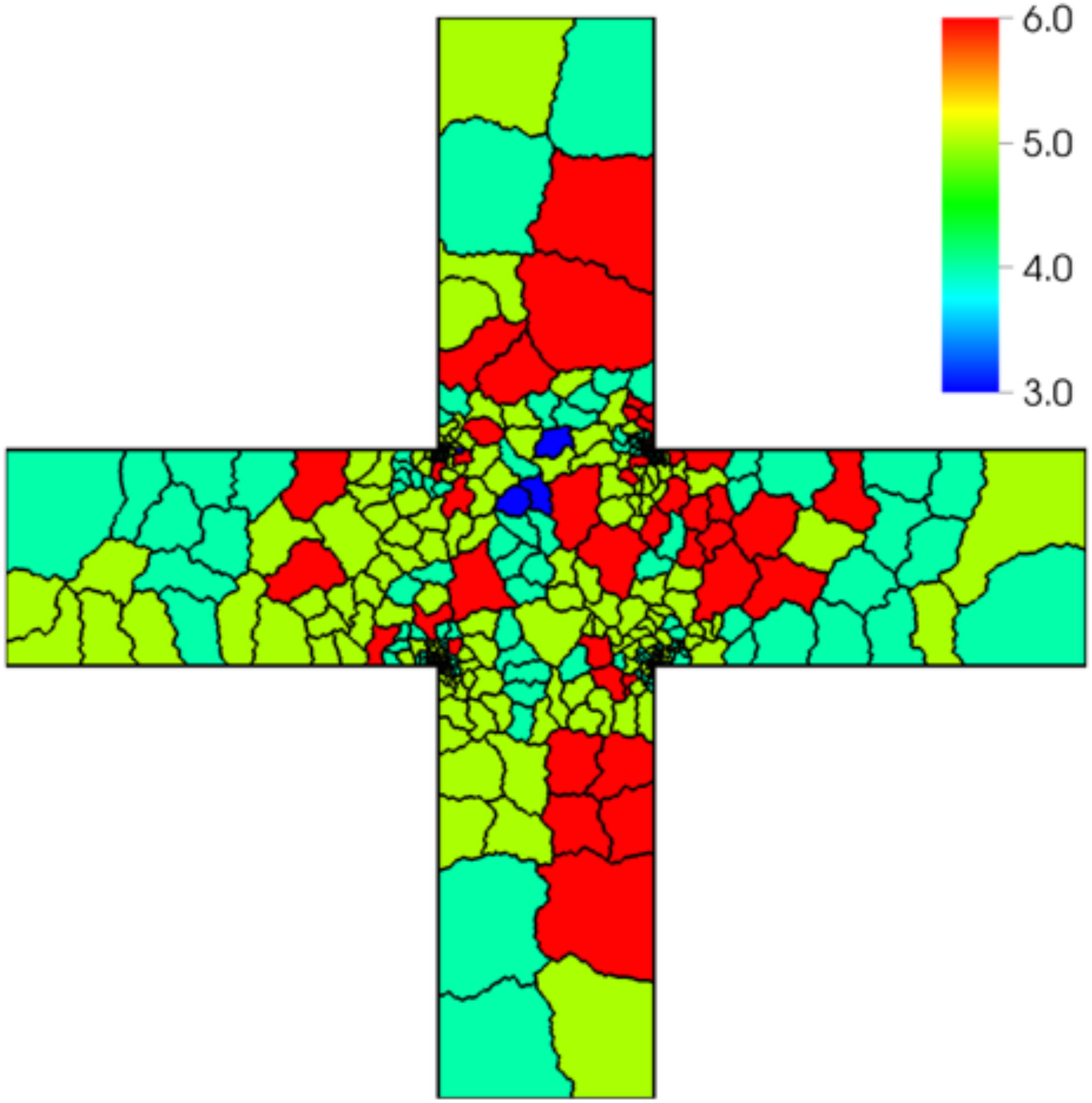


u_2

Cross Domain in 2D

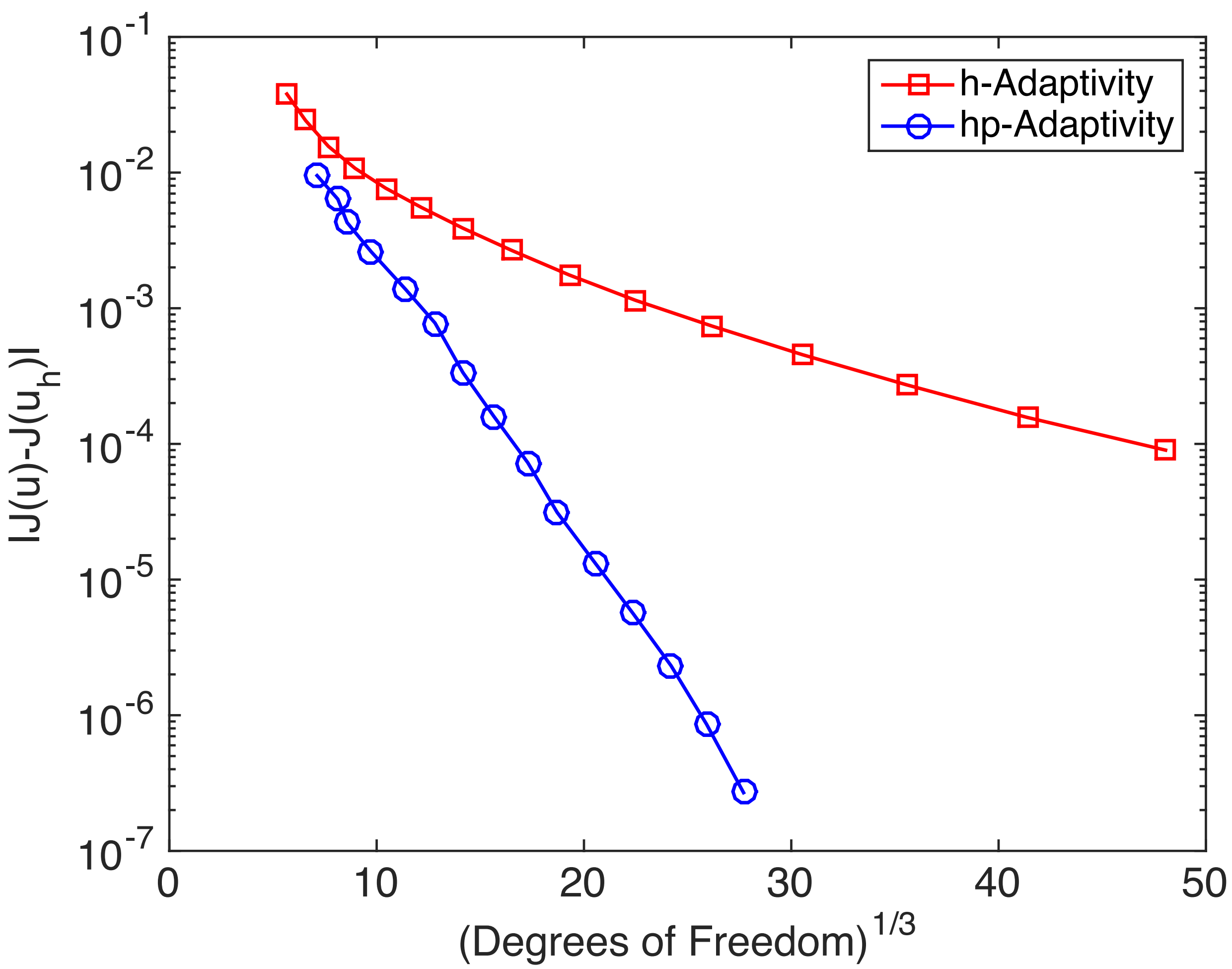


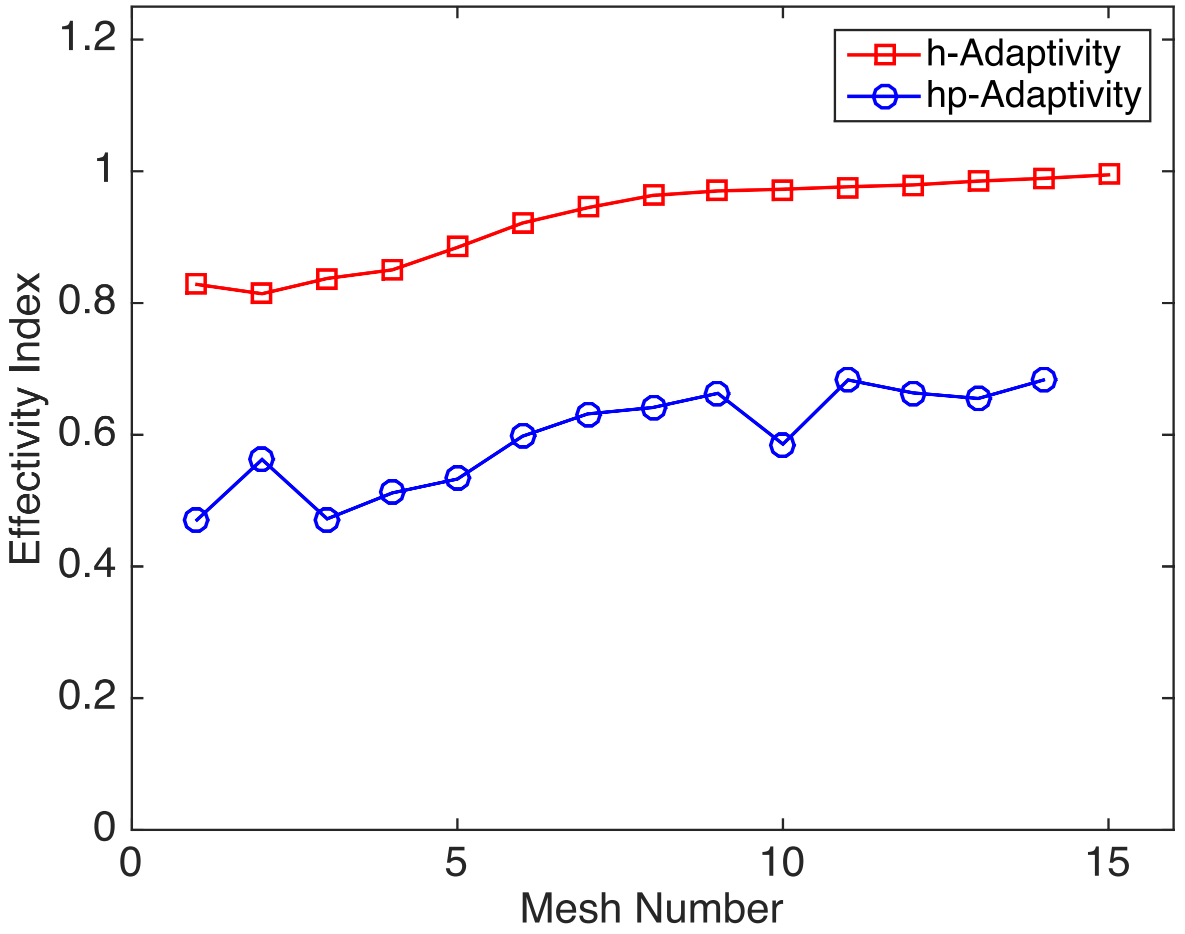
hp-Mesh after 9 refinements.



hp-Mesh after 14 refinements.

Cross Domain in 2D



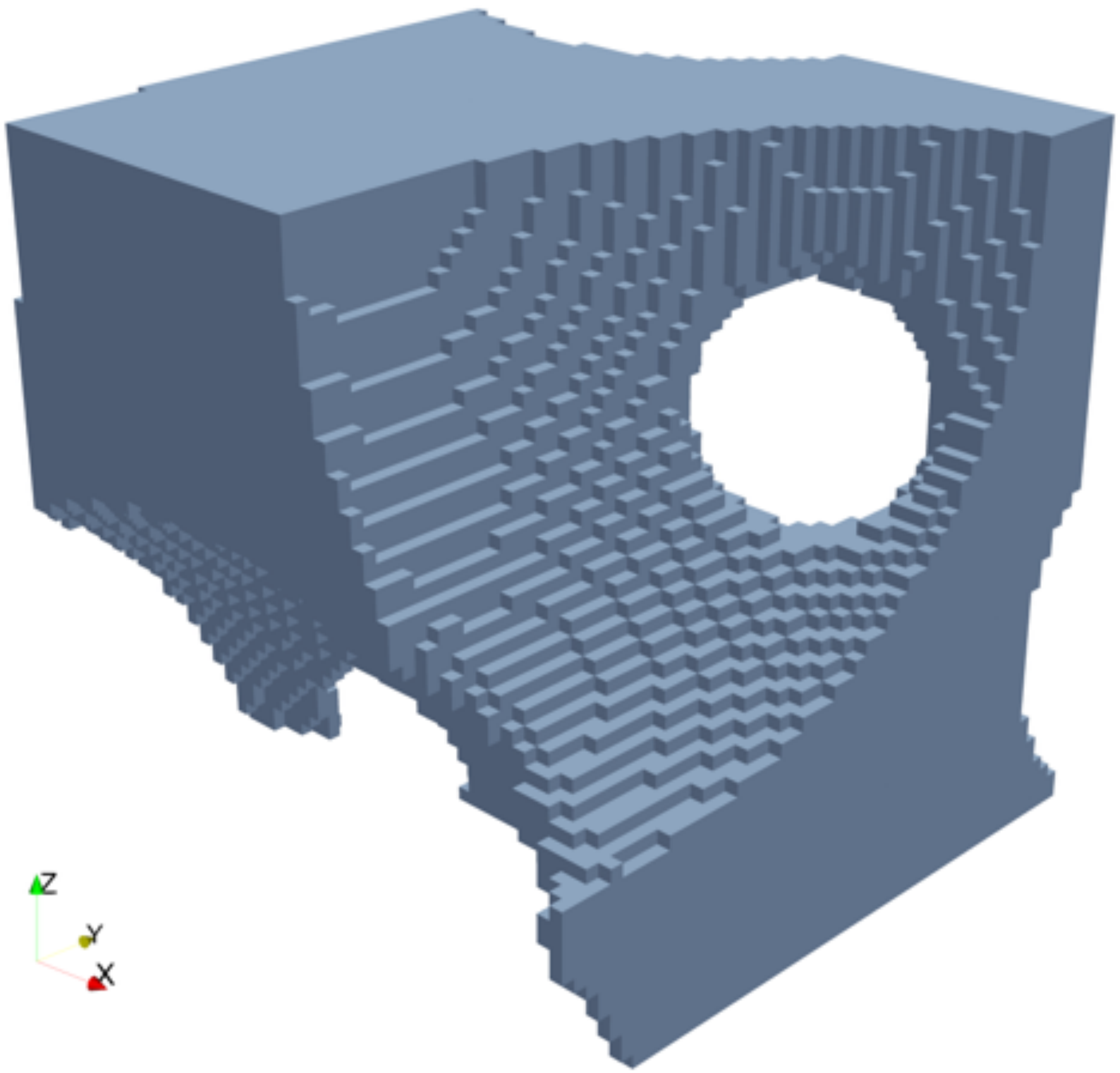


Permeability

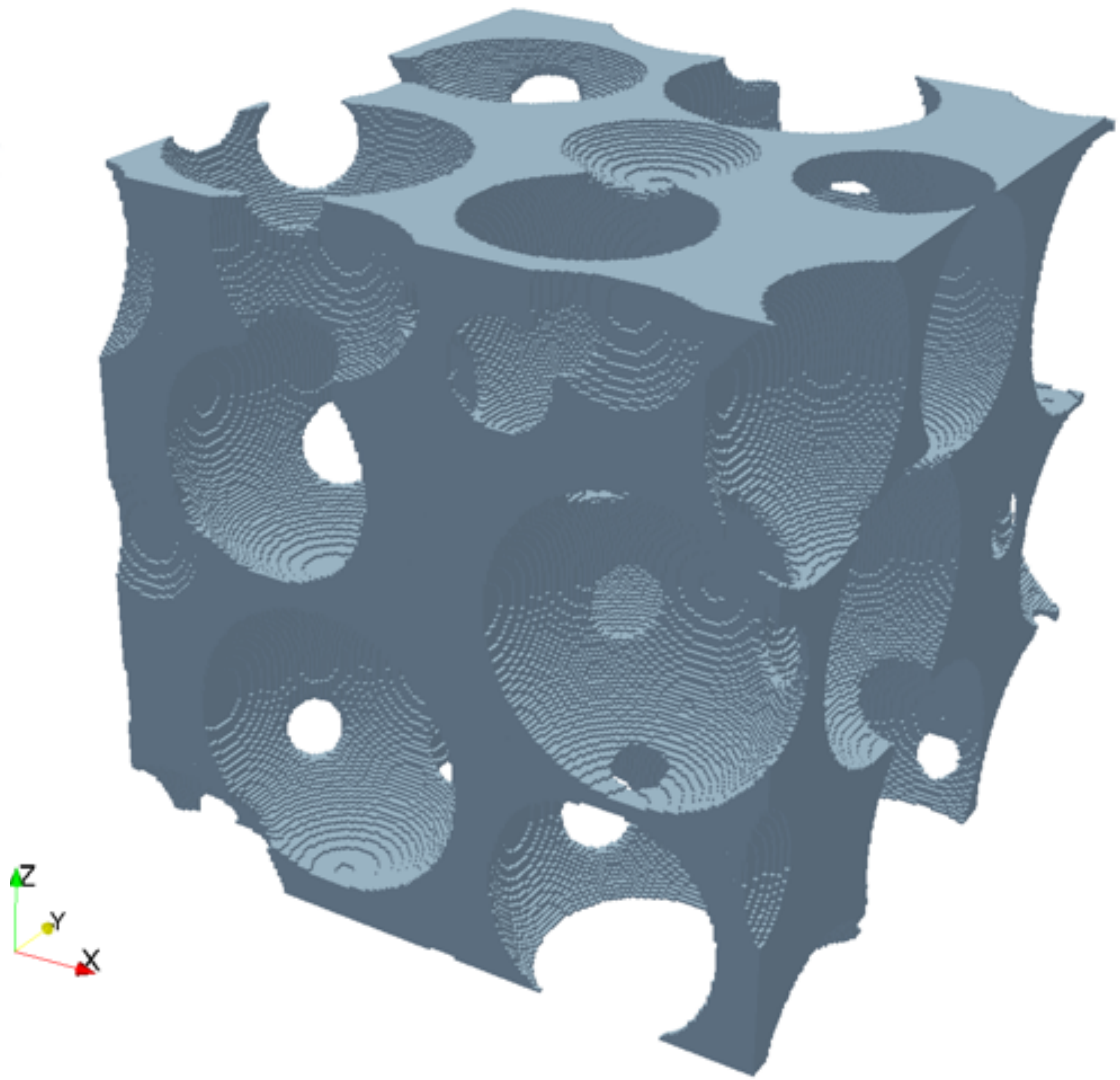
$$
\begin{aligned}
-\Delta u + \nabla p &= 0 && \text{in } \Omega, \\
\nabla \cdot u &= 0 && \text{in } \Omega, \\
u &= 0 && \text{on } \partial\Omega_{\text{wall}}, \\
(\nabla u - pI)n &= 1 && \text{on } \partial\Omega_{\text{inflow}}, \\
(\nabla u - pI)n &= 0 && \text{on } \partial\Omega_{\text{outflow}}.
\end{aligned}
$$

$$
\begin{aligned}
J(u,p) &= \frac{1}{\Delta p}\int_{\Omega} u_i \,\mathrm{d}x, \\
\Delta p &= \frac{1}{|\partial\Omega_{\text{inflow}}|}\int_{\partial\Omega_{\text{inflow}}} p \,\mathrm{d}s - \frac{1}{|\partial\Omega_{\text{outflow}}|}\int_{\partial\Omega_{\text{outflow}}} p \,\mathrm{d}s.
\end{aligned}
$$

Permeability

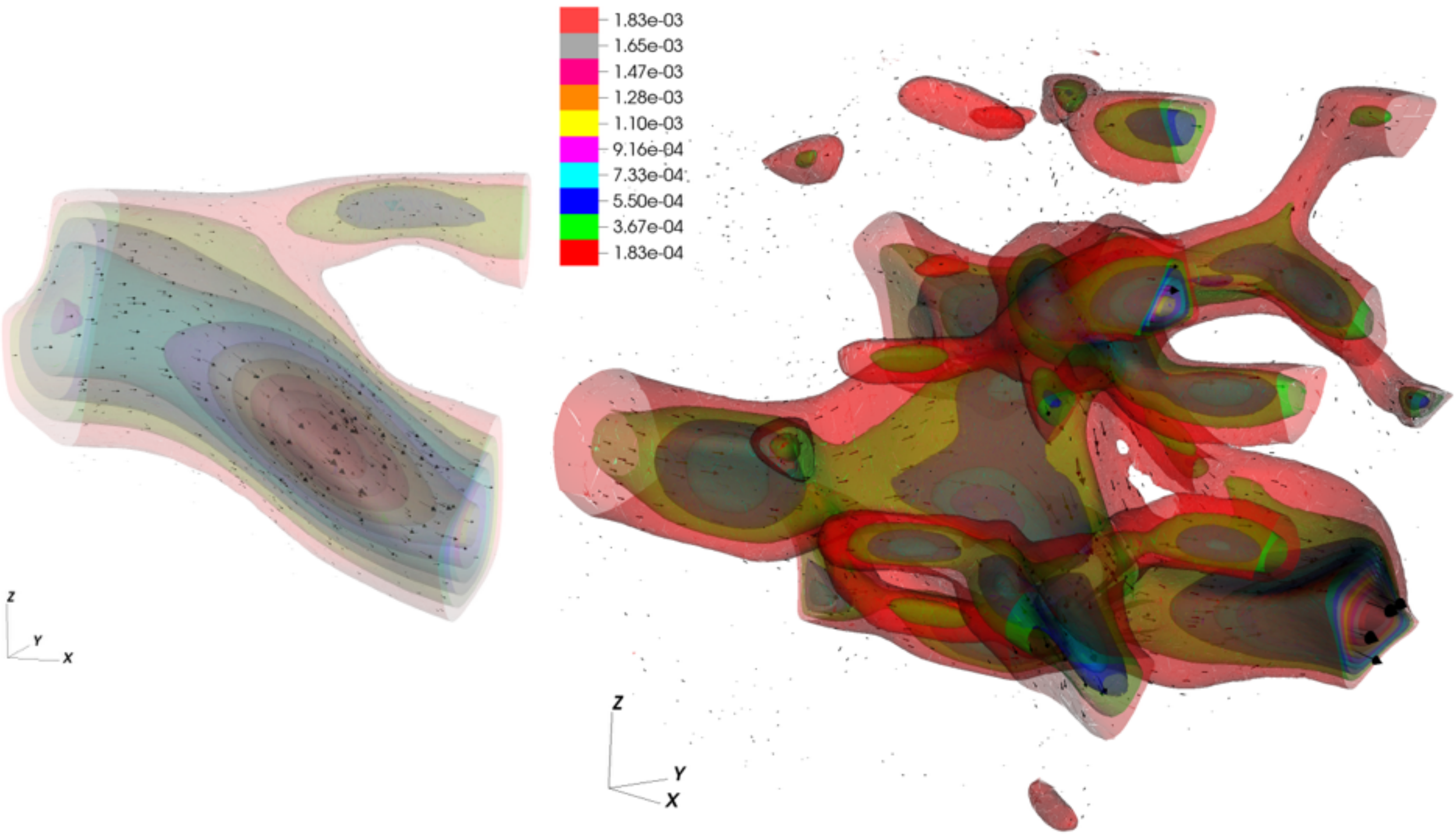


Coarse Sample



Fine Sample

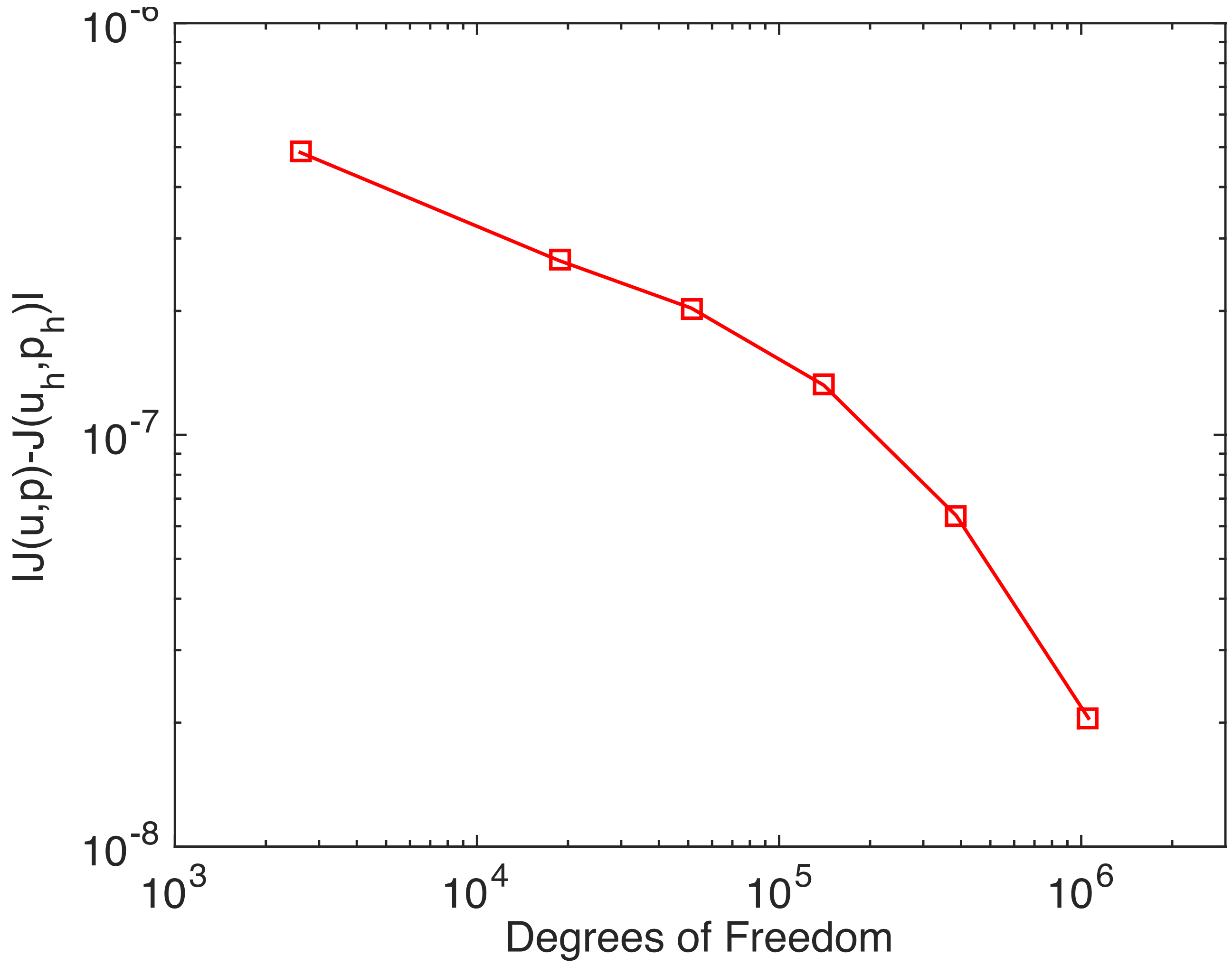
Permeability



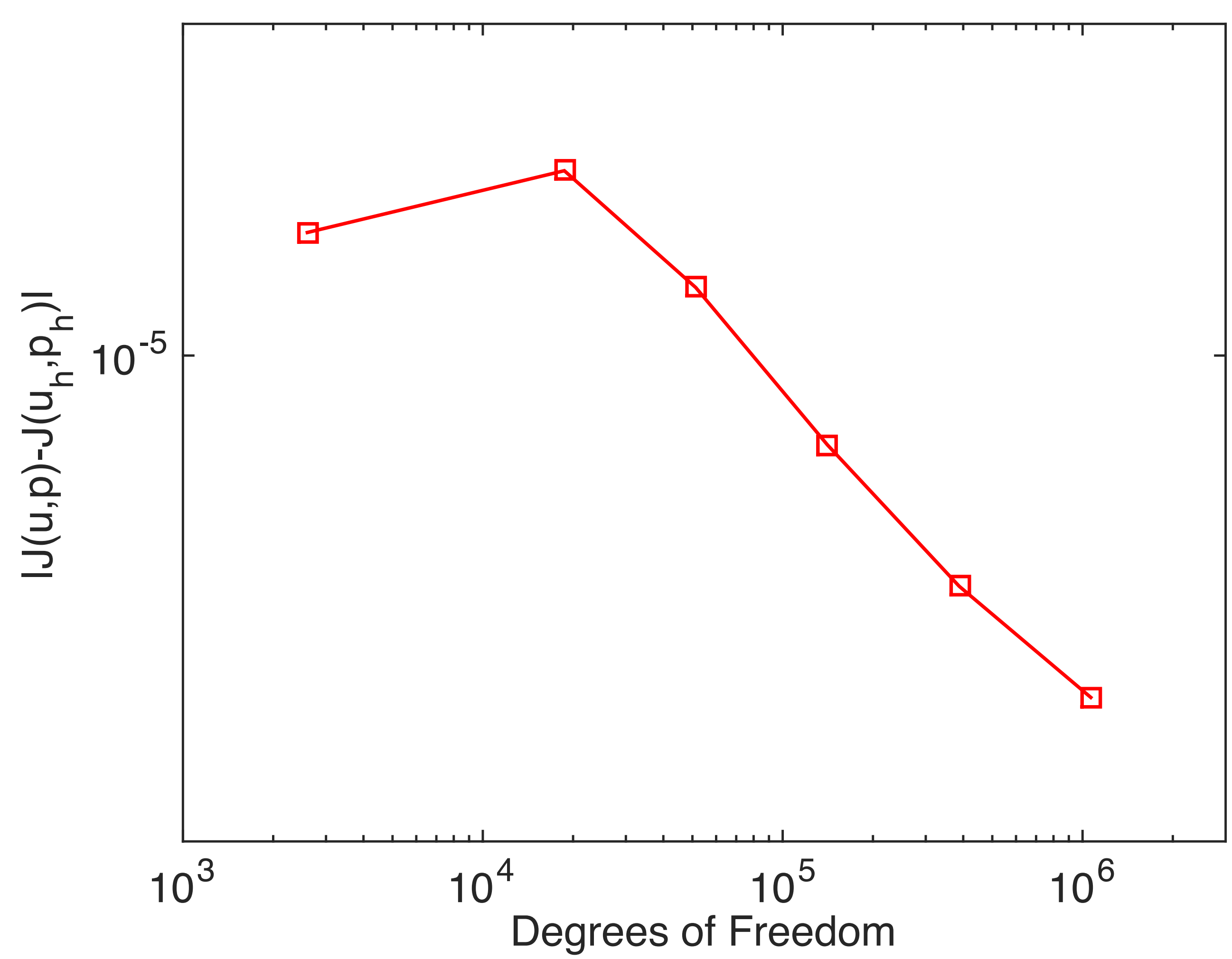
Coarse Sample

Fine Sample

Permeability

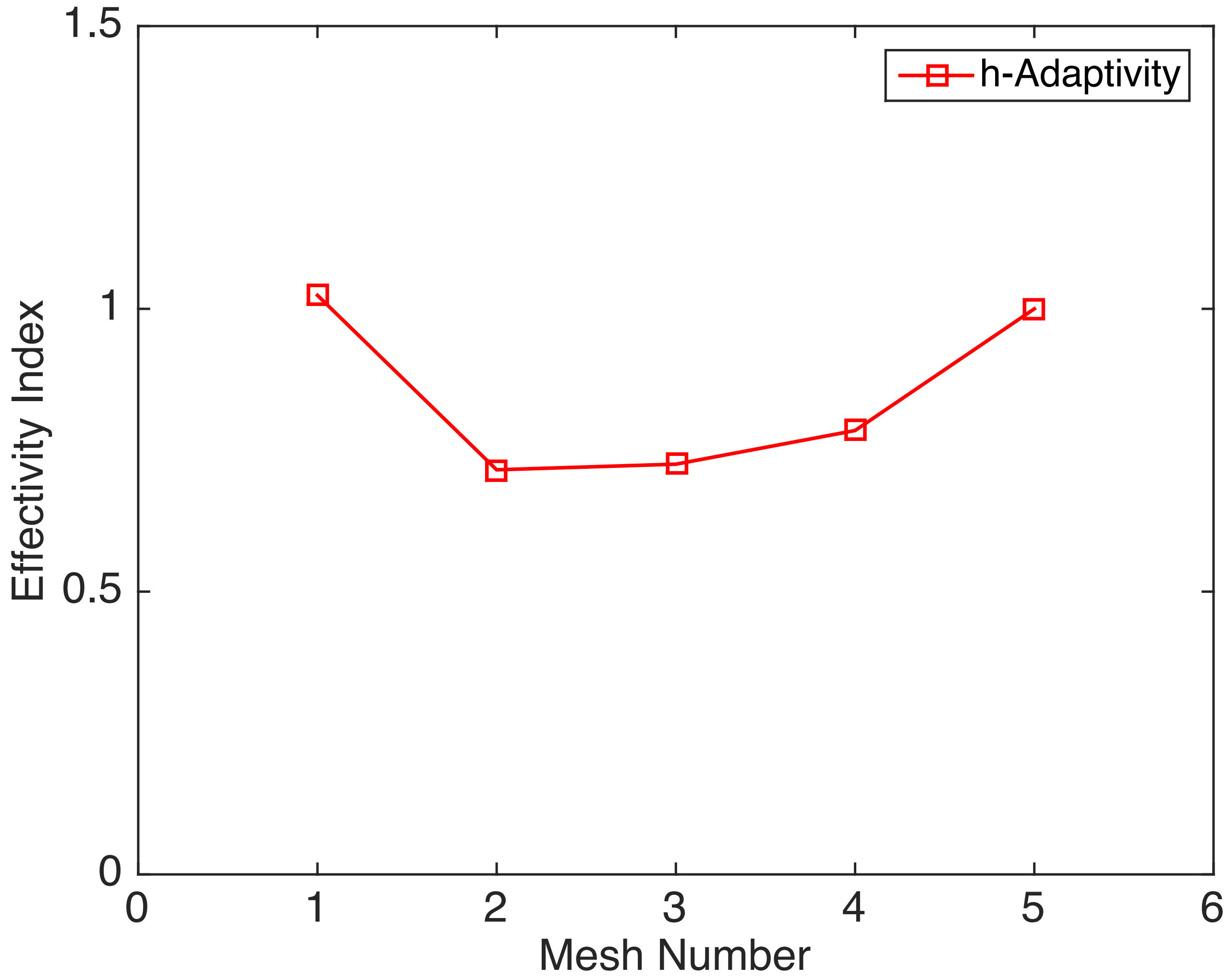


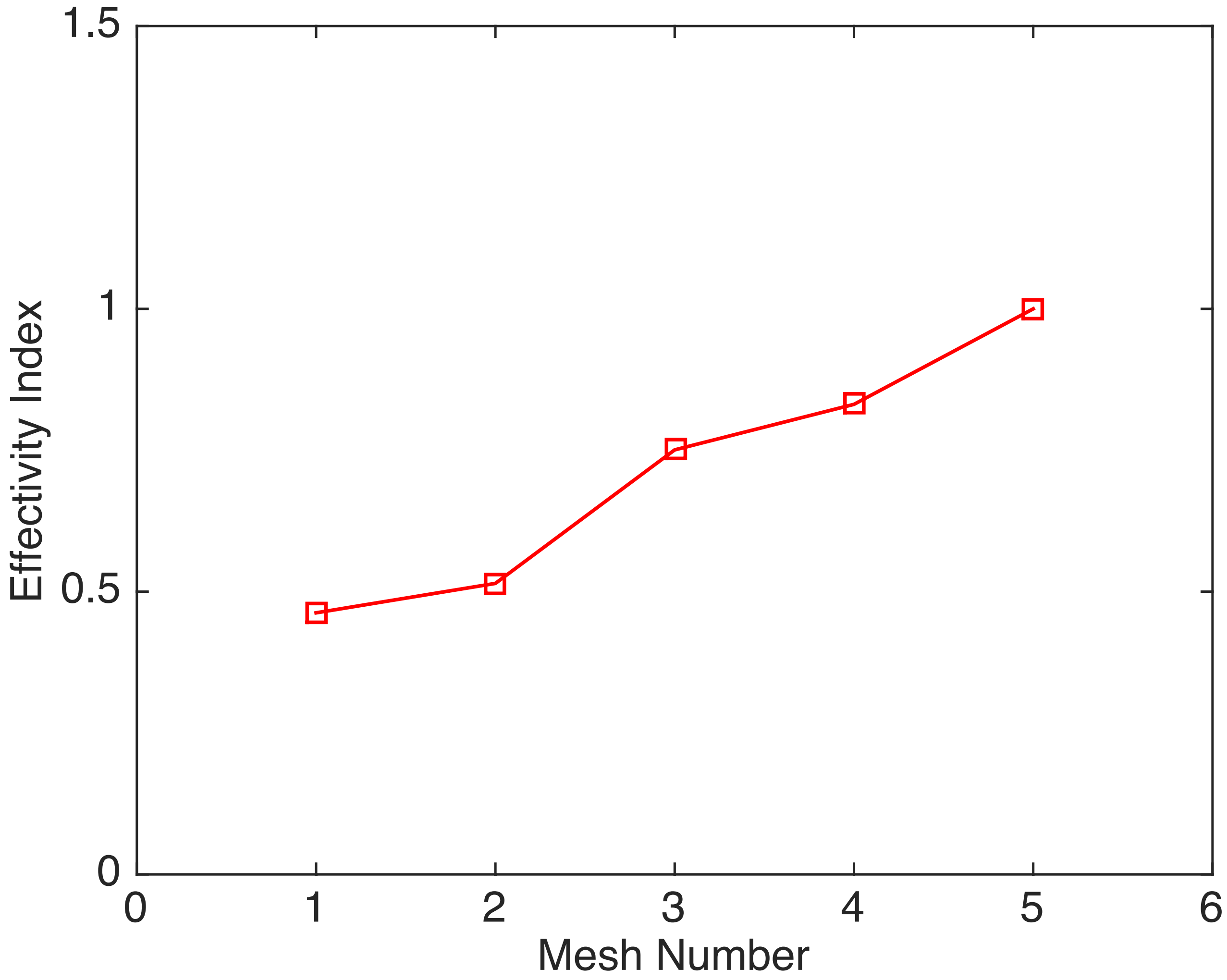
Coarse Sample



Fine Sample

Permeability





Coarse Sample

Fine Sample

Summary and Outlook

Summary and Outlook

- Developed the error analysis of DGFEMs on general polytopic meshes:
  - ☑ Number of degrees of freedom is *independent* of the domain;
  - ☑ Coarse approximations may be computed with engineering accuracy;
  - ☑ Adaptivity is focused on resolving *important features* of the solution;
  - ☑ Method naturally admits high-order polynomial orders;
  - ☑ May be exploited as coarse level solvers with multilevel solvers.
    DD: Antonietti, Giani, & H. 2014, Giani & H. 2014; MG: Antonietti, H., Sarti, & Verani 2014
- Analysis of DGFEMs on general polygonal/polyhedral meshes accounts for local edge/face degeneration.
- Development of multigrid preconditioners.
  Antonietti, H., Hu, Sarti, & Verani 2017
- Extension to multiscale problems, exploiting the HMM approach.
- Application to two-grid methods for nonlinear PDEs.
  Congreve, H., & Wihler 2011, 2013, Congreve & H. 2013
- Efficient Implementation: Geometry Handling/Smoothing, Quadrature.

References

1. P.F. Antonietti, A. Cangiani, J. Collis, Z. Dong, E.H. Georgoulis, S. Giani and PH, 2016. Review of Discontinuous Galerkin Finite Element Methods for Partial Differential Equations on Complicated Domains. In: G.R. Barrenechea, F. Brezzi, A. Cangiani and E.H. Georgoulis, eds., Building Bridges: Connections and Challenges in Modern Approaches to Numerical Partial Differential Equations: Lecture Notes in Computational Science and Engineering Springer International Publishing. 281-310.
2. A. Cangiani, Z. Dong, E.H. Georgoulis and PH, 2016. hp-Version Discontinuous Galerkin Methods for Advection-Diffusion-Reaction Problems on Polytopic Meshes ESAIM: M2AN. 50(3), 699-725.
3. J. Collis and PH, 2016. Adaptive Discontinuous Galerkin Methods on Polytopic Meshes. In: G. Ventura and E. Benvenuti, eds., Advances in Discretization Methods: Discontinuities, Virtual Elements, Fictitious Domain Methods Springer International Publishing. 187-206.
4. P.F. Antonietti, S. Giani and PH, 2014. Domain Decomposition Preconditioners for Discontinuous Galerkin Methods for Elliptic Problems on Complicated Domains J. Sci. Comp. 60(1), 203-227.
5. S. Giani and PH, 2014. hp-Adaptive Composite Discontinuous Galerkin Methods for Elliptic Problems on Complicated Domains Num. Meth. PDEs. 30(4), 1342-1367.
6. A. Cangiani, E.H. Georgoulis and PH, 2014. hp-Version discontinuous Galerkin methods on polygonal and polyhedral meshes M3AS. 24(10), 2009-2041.
7. P.F. Antonietti, S. Giani and PH, 2013. hp-Version Composite Discontinuous Galerkin Methods for Elliptic Problems on Complicated Domains SIAM J. Sci. Comp. 35(3), 1417-1439.