# Spectral theory for Schrödinger operator
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The main object of these lectures will be Schrödinger operators on Euclidean spaces or Riemannian manifolds. On $\mathbb{R}^n$, these operators are of the form

$$\mathcal{L} = -\sum_{j=1}^{n} \frac{\partial^2}{\partial x_i^2} - V$$

and are associated to the quadratic form :

$$u \mapsto \int_{\mathbb{R}^n} u\mathcal{L}u = \int_{\mathbb{R}^n} \left[ |\nabla u|^2 - Vu^2 \right].$$

We will begin by the spectral study of these operators, including the Friedrichs extension and the description of an general framework : Dirichlet spaces. We will spent sometimes on examples : weighted Laplacian, magnetic Laplacian, Schrödinger operator with singular potential. We will then study more refined properties of the spectrum of such operator : essential and discrete spectrum, non negative eigenvalue and we will give a proof of the Cwickel-Rosenblum estimate of the number of negative eigenvalue of Schrödinger operators on Euclidean space $\mathbb{R}^{n>2}$, , this number is bounded from above by

$$C(n) \int_{\mathbb{R}^n} V_+^{\frac{n}{2}}.$$

We will then describe more precisely the case of non negative Schrödinger operators and will study the equation $\mathcal{L}u = f$ and will give several characterization of the existence of a positive fundamental solution for this equation i.e. the existence of positive Green kernel $G(x, y)$ solution of

$$\mathcal{L}_y G(x, y) = \delta_x.$$

We will finish by some application of Agmon's type estimate to rigidity result in geometric analysis.

## References


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