Minicourses:

- **Semyon Dyatlov and Maciej Zworski: Microlocal methods in chaotic dynamics**

  Following the insights of Faure–Sjoestrand and Tsujii, microlocal/semiclassical methods have proved themselves very useful in the study of closed and open smooth hyperbolic systems. We will explain how they give a simple proof of the meromorphy of Ruelle zeta function for Anosov flow (and indicate the ideas behind the Axiom A case), show stochastic stability of Ruelle resonances, and explain the order of vanishing of the zeta function at zero in the case of negatively curved surfaces (showing in particular that the length spectrum determines the genus).

- **Livio Flaminio: $\mathrm{SL}_2(\mathbb{R})$: harmonic analysis and dynamics**

  The group $\mathrm{SL}_2(\mathbb{R})$ is at the cross-road of several lines of mathematical development: for number theorists its arithmetical lattices are objects of fundamental interest; in dynamics, it provides the simplest example of mixing Anosov flow; for harmonic analysts $\mathrm{SL}_2(\mathbb{R})$ is the starting point of the harmonic analysis of reductive Lie groups.

  In this course, we will review the geometrical and analytic properties of $\mathrm{SL}_2(\mathbb{R})$, introduce its representation theory and its applications to hyperbolic surfaces, thereby providing a fundamental example of the theory of resonances for hyperbolic flows developed in the parallel courses.

- **Carlangelo Liverani: Functional analytic approach to dynamical systems**

  I will discuss the study of the statistical properties of dynamical systems from a functional analytic point of view.

  Lecture 1: The basic idea: expanding maps. Synopsis: I will discuss how to study statistical properties and limit theorems for the simple case of smooth expanding maps.

  Lecture 2: Hyperbolic systems and anisotropic Banach spaces. Synopsis: I will discuss how to upgrade the strategies put forward in the previous lecture to the case of hyperbolic dynamics. To do so it is necessary to introduce unconventional functional spaces. Such spaces came in various form, I will discuss the so called "geometric spaces".

  Lecture 3: An application to the case of parabolic dynamics. Synopsis: I will discuss how parabolic dynamics can be treated via renormalization techniques. In the case in which the renormalising dynamics is hyperbolic, the results discussed in the previous lectures can be used to obtain results on statistical properties of the parabolic system.

Talks:

- **Yves Colin de Verdière: On the dynamics of internal waves in a domain with topography**

This is a work in progress with Laure St-Raymond (ENS Lyon) and Frédéric Faure (IF).

Stratification of the density in an incompressible fluid is responsible for the propagation of internal waves. In domains with topography, these waves exhibit interesting properties. In particular, numerical and lab experiments performed in the team of Thierry Dauxois, especially by Christophe Brouzet, at ENS Lyon, show that in 2D these waves concentrate on attractors for some generic frequencies of the forcing.

I will present simplified mathematical model and discuss some of their properties.

- **Frédéric Faure: A semi-classical analysis with wave packets for the Ruelle-Pollicott spectrum of hyperbolic dynamics**

  Uniformly hyperbolic dynamics (Anosov, Axiom A) have "sensitivity to initial conditions" and manifest "determinist chaotic behavior", e.g. mixing, statistical properties etc. The generator of the evolution operator (the "transfer operator") has a discrete spectrum, called "Ruelle-Pollicott resonances" which describes the effective convergence and fluctuations towards equilibrium. We will present a method of analysis using decomposition into wave packets (or wavelets) that gives new results. This method is similar to the "Weyl-Hörmander phase-space metric method". We will also discuss numerical computation of the spectrum. Joint work with Masato Tsujii.

- **Frédéric Naud: Large covers and resonances of hyperbolic surfaces**

  In this talk, after a brief refresher on the spectral theory of infinite area surfaces, we will investigate the behaviour of resonances for families of Galois covers of a given surface when the degree goes to infinity. In particular we will prove a "Weyl Law" for large covers, and an existence result of non-trivial sharp resonances in the case of congruence subgroups.

- **Françoise Pène: Stochastic properties of the $\mathbb{Z}^2$-periodic Sinai billiard**

  We study some stochastic properties of the $\mathbb{Z}^2$-periodic Sinai billiard. We investigate the following properties of this dynamical system preserving an infinite measure: recurrence, ergodicity, mixing, decorrelation, limit theorems.

- **Mark Pollicott: Rigorous bounds in computing dynamical invariants**

  In the context of analytic dynamical systems it is possible to use zeta fuctions and determinants to give good numerical estimates on associated values, such as: Hausdorff Dimension of Limit sets; Entropy and Lyapunov exponents of hyperbolic systems; variance, etc. We will discuss the

method, illustrating it with specific examples, concentrating on new rigorous bounds on the errors in the approximation. This is joint work with Oliver Jenkinson.

- **Gabriel Rivière: Spectral analysis of Morse-Smale gradient flows**

  Given a smooth function and a Riemannian metric on a compact and boundaryless manifold, one can define a gradient vector field. Under generic assumptions (of Morse-Smale type), I will explain how one can compute explicitely the spectrum of the associated Lie derivative acting on certain anisotropic Sobolev spaces of currents. I will also discuss the relation of this spectral analysis with differential topology. This is a joint work with Nguyen Viet Dang (University Lyon 1).

- **Masato Tsujii: Exponential decay of correlations for Anosov flows**

  In this talk, I would like to present a result on exponential decay of correlations for generic volume-preserving Anosov flows on 3-dim manifolds. The main ingredient of the proof is a simple observation on local geometric structure of the strong unstable foliation. If time allows, I would like to discuss about generalization of such an observation to higher dimensional cases.

- **Tobias Weich: Numerical calculation of resonances on convex-cocompact surfaces via zeta-functions**

  Convex-cocompact surfaces are a certain class of noncompact constant curvature surfaces, that can be obtained by taking the quotient of the upper half-plane with respect to a certain discrete subgroup of PSL(2,R). Thanks to their strong geometric and algebraic structures, these surfaces are an ideal class of geometries to study of resonances of the geodesic-flow transfer operator as well as resonances of the Laplacian. Concerning the distribution of those resonances there have been established a large number of interesting results and conjectures during the past years.

  In this talk we will explain how the resonances on convex-cocompact surfaces can be efficiently calculated numerically. This allows to test existing conjectures as well as to discover new interesting structures in the resonances spectrum. The method of these numerical calculations is entirely based on approximating dynamical zeta functions and uses techniques developed by Jenkinson-Pollicott and in joint work with David Borthwick.