Workshop - Simulation and probability: recent trends

Program

Tuesday 5th

10h30 - 11h30 Welcome
11h30 - 12h30 S. Herrmann

12h30 - 14h00 Lunch

14h00 - 15h00 N. Berglund
15h00 - 16h00 H. Touchette
16h00 - 16h30 Break
16h30 - 17h30 Discussions

Wednesday 6th

9h00 - 11h00 P. Dupuis
11h00 - 11h30 Break
11h30 - 12h30 G. Fort

12h30 - 14h00 Lunch

14h00 - 15h00 A. Lee
15h00 - 16h00 F. Bouchet
16h00 - 16h30 Break
16h30 - 17h30 N. Kantas

Thursday 7th

9h00 - 11h00 P. Dupuis
11h00 - 11h30 Break
11h30 - 12h30 J. Reygner

12h30 - 14h00 Lunch

14h00 - 16h00 D. Crisan
16h00 - 16h30 Break
16h30 - 17h30 V. Lecomte

19h00 - Dinner (invited speakers only)

Friday 8th

9h00 - 11h00 D. Crisan
11h00 - 11h30 Break
11h30 - 12h30 J. Tugaut

Nils Berglund

Metastable Markov chains, trace process and spectral theory

I will consider discrete-time Markov chains, either in discrete or in continuous space, which are metastable in the sense that they spend long time spans in relatively small subsets of space. These chains are not assumed to be reversible. The trace process associated with a subset A of space is the Markov chain monitored only when staying in A. Together with related Laplace transforms, this process turns out to be extremely useful to uncover the metastable dynamics of the chain, compute convergence rates to equilibrium, obtain spectral information, and prove results on approximation by Markov chains with fewer states. I will illustrate this approach on a number of simple examples.

Joint work with Manon Baudel (CERMICS, Ecole des Ponts).

Freddy Bouchet

Rare events and instantons in complex dynamical systems: the examples of climate and the solar system dynamics

I will discuss the phenomenology of rare events through examples coming from deterministic systems: climate dynamics and the solar system dynamics. I will discuss when and why rare events in those very complex chaotic systems have the same phenomenology as the ones in stochastic differential equations. How to compute those rare events is another key question. I will question the conditions for current rare event algorithms to be useful.

Dan Crisan

Particle Filters in High Dimensions

Particle filters are a set of probabilistic algorithms used to solve filtering problems arising in signal processing and Bayesian statistical inference. Their area of applicability is currently being extended to solve high dimensional problems such as those encountered in data assimilation problems for numerical weather prediction. The lectures will contain an elementary introduction to particle filters with emphasis on their applicability to such problems. I will discuss the specific difficulties encountered when applying particle filters to high dimensional problems as well as procedures required for their successful implementation. Depending on the time, I will cover: model reduction (high to low resolution), tempering, jittering, uncertainly quantification, initialization, nudging and localization. The running example covered in the lectures will be an application to a partially observed solution of a damped and driven incompressible 2D Euler equation with stochastic advection by Lie transport (further details of the example can be found in
https://arxiv.org/abs/1801.09729)

The lectures are based on joint work with Wei Pan (Imperial College London)

Paul Dupuis

Monte Carlo methods for the approximation and mitigation of rare events

This presentation consists of two talks on different but related problems involving rare events. The first problem is the estimation of a probability or expected value which is largely determined by whether or not a rare event occurs. The second problem is overcoming what is sometimes called the rare event sampling problem, which refers to the difficulties of computing integrals with respect to the stationary distribution of a Markov process for which parts of the state space do not communicate well. The perspective taken for both problems is to use large deviation theory to help in problems of performance analysis and design. In the context of the first problem, after reviewing methods often used for rare event simulation (importance sampling and particle splitting), we outline how sample path large deviation theory, and in particular the form of the rate function, can be used to analyze and characterize schemes that perform well and also optimally according to certain criteria. Associated with both methods is an importance function, and large deviation theory is used to characterize critical properties of this function. We will point out qualitative similarities and differences between the methods, and also describe recent results concerning non-asymptotic bounds and methods suitable for large time problems near metastable points. In the context of the second problem we first review parallel tempering (also known as replica exchange), which is a method that is widely used in the physical sciences and elsewhere for computations with respect to a Gibbs distribution. We then present several results on large deviation properties of parallel tempering and a related process known as infinite swapping that is obtained as the exchange rate tends to infinite. Large deviation approximations for both large time and low temperature limits are used to extract interesting and useful information on the numerical method, and applications to problems from the physical sciences and counting problems will be presented.

Gersende Fort

Convergence and Efficiency of Adaptive Importance Sampling techniques with partial biasing

Usual Monte Carlo algorithms have an extremely poor behavior when sampling multimodal target distributions. We propose a robust method, which is among the family of adaptive importance sampling techniques; this algorithm is a generalization of the discrete-time Self Healing Umbrella Sampling method. The importance function is based on the weights of disjoint sets which form a partition of the space. In the context of computational statistical physics, the logarithm of these weights is, up to a multiplicative constant, the free energy, and the discrete valued function defining the partition is called the reaction coordinate. The algorithm is a generalization of the original Self Healing Umbrella Sampling method in two ways: (i) the updating strategy leads to a larger penalization strength of already visited sets and (ii) the target distribution is biased using only a fraction of the free energy, in order to increase the effective sample size and reduce the variance of importance sampling estimators. When analyzing its behavior, we read this new algorithm as a Stochastic Approximation algorithm, with random stepsizes, and fed with Adaptive Markov Chain Monte Carlo samples. In this talk, we will discuss the role of the design parameters through numerical illustrations on a toy example; and will give convergence results of both the sampling technique and the weight estimators.

This talk is based on the paper "Convergence and efficiency of adaptive importance sampling techniques with partial biasing", by G. Fort, B. Jourdain, T. Lelivre, G. Stltz, and published in Journal of Statistical Physics.

Samuel Herrmann

Simulation of Brownian motion's exit time from a domain

First I will present algorithms which permit to simulate the first-passage time (FPT) of the Brownian motion to a curved boundary (one-dimensional problem) and by extension to deal with one-dimensional diffusions FPT. The main interest of these methods is that they avoid splitting time schemes as well as inversion of complicated series. These algorithms are based either on an iterating procedure or on an acceptance-rejection method. The second part of the talk shall focus on the higher dimensional case: the simulation of the exit time and position of a d-dimensional Brownian motion from a domain. We build a fast and accurate numerical scheme for approximating the exit time and present an application to the Initial-Boundary Value Problem for the heat equation.

This is joint work with M. Deaconu (INRIA), S. Maire (University of Toulon), E. Tanr (INRIA), C. Zucca (University of Torino).

Nikolas Kantas

Particle Filtering for Stochastic Navier-Stokes Signal Observed with Additive Noise

Traditional particle filtering methodology has been extremely successful in low dimensional non-linear non-Gaussian applications (e.g Doucet et. al. 01), but their application in high dimensional settings has been very challenging partly due to the difficulty to perform importance sampling efficiently in high dimensions (Snyder et. al. 08, Bengtsson et. al. 08, Bui Quang et. al. 10). Despite this challenge a few successful high dimensional particle filtering implementations have appeared recently for data assimilation applications when the hidden signal obeys discrete time dynamics (Chorin et. al. 09, Papadakis et. al. 10, Reich 13, Van Leeuwen 10, 12, Weare 09). In this talk we present our work for addressing problems where the signal of interest obeys continuous time dynamics and in particular is modelled by the stochastic Navier Stokes in 2D that is observed at discrete times with additive noise. The setup is relevant to data assimilation for numerical weather prediction and climate modelling, where similar models are used for unknown ocean or wind velocities. We will present a particle filter that uses adaptive tempering (like Jasra et. al. 11), likehood informed importance proposals (similar to Golightly and Wilkinson 08), and pre-conditioned Crank Nicholson MCMC steps (similar to Hoang et al 14, Cotter et al 13). We will show some numerical results that demonstrate the necessity of each step in terms of achieving good performance and efficiency.

This is joint work with Francesc Pons-Llopis (Imperial), Alex Beskos (UCL), Ajay Jasra (NUS).

Vivien Lecomte

Population dynamics method for rare events: systematic errors and feedback control

We analyze large deviations of the time-averaged activity in the one-dimensional Fredrickson-Andersen model, both numerically and analytically. The model exhibits a dynamical phase transition, which appears as a singularity in the large deviation function. We analyze the finite-size scaling of this phase transition numerically, by generalizing an existing cloning algorithm to include a multicanonical feedback control: this significantly improves the computational efficiency. Motivated by these numerical results, we formulate an effective theory for the model in the vicinity of the phase transition, which accounts quantitatively for the observed behavior. We discuss potential applications of the numerical method and the effective theory in a range of more general contexts.

Work in collaboration with Takahiro Nemoto, Robert L. Jack

Anthony Lee

Particle filters and variance estimation

Particle filters, or sequential Monte Carlo methods, are random algorithms for approximating certain types of integrals that arise in the analysis of data. I will present new variance estimators for the resulting approximations that can be computed using a single run of the algorithm. This builds upon advances on the one hand by Chan and Lai, who proposed the first variance estimator of this form, and by Cérou, Del Moral and Guyader, who derived the first non-asymptotic second moment expressions for particle filter approximations. As the number of particles grows, the variance estimators we propose are weakly consistent for asymptotic variances of the Monte Carlo approximations and some of them are also non-asymptotically unbiased. The asymptotic variances can be decomposed into terms corresponding to each time step of the algorithm, and we show how to estimate each of these terms consistently.

Julien Reygner

Eyring-Kramers formula for nonreversible diffusion processes

We will present formal arguments and computations leading to the derivation of a generalisation to the nonreversible case of the Eyring-Kramers formula for the prefactor of mean transition times of bistable diffusion processes. In this formula, a nonlocal term appears, which involves the Hessian of the quasipotential along the instanton, and vanishes as soon as the stationary distribution of the process is a Gibbs measure. We will also discuss a numerical method to compute this term, based on matrix Riccati equations. These are joint works with Freddy Bouchet.

Hugo Touchette

Large deviation simulations: Reversible vs nonreversible processes

I will give in this talk a brief overview of different numerical approaches, based on spectral theory, importance sampling and control theory, that can be used to estimate large deviation functions characterizing the fluctuations of time-integrated functionals of Markov processes. In discussing these methods, I will draw attention to simplifications and difficulties that arise when applying them to reversible or non-reversible processes.

Julian Tugaut

Exit time of a self-stabilizing diffusion in non convex landscape

And application to the characterization of the basins of attraction of the granular media equation

In this talk, we briefly recall some results of the Freidlin-Wentzell theory then we give a Kramers'law satisfied by the McKean-Vlasov diffusion when the confining potential is uniformly strictly convex. We briefly present two previous proofs of this result then we give a third one which is simpler, more intuitive and less technical. Finally, we give some ideas to obtain the Kramers'law when the confining potential is not convex. And, if there is enough time, we will give the idea to use this method in order to characterize the basins of attraction of the granular media equation in the case in which there are three invariant probabilities.