ABSTRACTS

Olivier Benoist : Sums of few squares in real-analytic geometry.

Pfister has obtained a quantitative version of Hilbert's 17th problem: a nonnegative real polynomial in n variables is a sum of 2^n squares of rational functions. We will explain an analytic analogue of this theorem, which in particular implies that a nonnegative real-analytic function on a compact real-analytic manifold of dimension n is a sum of 2^n squares of real-analytic meromorphic functions.

François Bernard : Locally Lipschitz rational functions on complex algebraic varieties.

Inspired by the work of Pham and Teissier about locally Lipschitz meromorphic functions on complex analytic varieties, we use the recent researches on complex regulous functions to study locally Lipschitz rational functions on complex algebraic varieties. As in the analytic case, we prove that locally Lipschitz rational functions correspond to the regular functions on an algebraic variety called the "Lipschitz saturation" which is obtained by an algebraic process. We deduce from this result conditions for two complex algebraic varieties to be birationally and locally bilipschitz equivalent.

Anna Bot : Survey on real forms.

In the last few years, there has been considerable progress in the understanding of how many nonisomorphic real forms a given complex variety can have. In the projective case, a variety can have at most countably many nonisomorphic real forms, where Lesieutre found the first example with infinitely many, and Dinh, Oguiso and Yu found a rational surface with infinitely many. No such restriction on the cardinality exists for affine or quasi-projective varieties, with examples of surfaces having uncountably many nonisomorphic real forms I constructed recently. In fact, in these examples, the real forms can be parametrised by a variety, which was further explored in joint work with Adrien Dubouloz. In this talk, I will survey the known results about real forms, and discuss some open problems.

Erwan Brugallé : Bitangents to real plane algebraic curves.

The study of bitangents of complex plane algebraic curves is a classical subject, and their number is prescribed by the Plüker formula. Over the real the situation remains widely unexplored, mainly due to the lack of methods to tackle this problem. In this talk I will survey what I know about real bitangents of real algebraic curves, and report on some recent work joint with Cristhian Garay and Thomas Blomme.

Jean-Baptiste Campesato : Motivic, logarithmic, and topological Milnor fibrations.

We compare the topological Milnor fibration and the motivic Milnor fibre of a regular complex function with only normal crossing singularities by introducing their common extension: the complete Milnor fibration for which we give two equivalent constructions. The first one extends the classical Kato-Nakayama log-space, and the second one, more geometric, is based on a the real oriented version of the deformation to the normal cone. In particular, we recover the topological Milnor fibration by quotienting the motivic Milnor fibration with suitable powers of $(0, \infty)$. Conversely, we also show that the stratified topological Milnor fibration determines the classical motivic Milnor fibre. (joint work with Goulwen Fichou and Adam Parusiński)

Adrien Dubouloz: Versal families of real forms.

A classical description due to Borel and Serre provides a set-theoretic one-to-one correspondence between isomorphism classes of real forms of a given real quasi-projective variety and cohomology classes of Galois 1-cocycles with values in the automorphism group of its complexification. The aim of the talk will be to motivate and present the essential elements of the construction of a refinement of this correspondence which allows to give one possible precise sense to the intuitive notion of a family of real forms of a given real variety depending algebraically on some real parameters. More precisely, I will explain the principle of the construction for any real scheme X of a sort of canonical "versal family of real forms" of X over a certain real form of the automorphism group functor of X whose real points are in natural one-to-one correspondence with the elements of the set of Galois 1-cocycles with value in the automorphism group of the complexification of X, and then, illustrate some properties of this versal family depending on the representability properties of the automorphism group functor of X. Based on a joint work with Anna Bot (University of Basel).

Lorenzo Fantini : Lipschitz geometry of complex surfaces.

Lipschitz geometry is a branch of singularity theory that studies a complex analytic germ $(X, 0)$ in $(C^n, 0)$ by equipping it with either one of two metrics: its outer metric, induced by the euclidean metric of the ambient space, and its inner metric, given by measuring the length of arcs on $(X, 0)$. Whenever those two metrics are equivalent up to a bi-Lipschitz homeomorphism, the germ is said to be Lipschitz normally embedded (LNE). I will give an overview of several results obtained together with André Belotto, András Némethi, and Anne Pichon on the Lipschitz geometry of surfaces, and more precisely on their inner metric structure, properties of LNE surfaces, and the so-called problem of polar exploration, which is the quest of determining the generic polar curves of a complex surface from its topology.

Olivier de Gaay Fortman: Abelian varieties as direct summands in Jacobians.

It is well-known that every complex abelian variety is dominated by the Jacobian of a smooth projective curve. However, it is not clear whether you can always find such a surjection which is split. Is every abelian variety a direct summand in a Jacobian variety? I will discuss this question in my talk, which turns out to be intimely related to the integral Hodge conjecture. This is joint work with Stefan Schreieder.

Johannes Huisman : Fundamental classes in bigraded cohomology.

We define a fundamental class of a subvariety of a smooth real algebraic variety with values in the bigraded cohomology of the latter. We show that it refines the equivariant fundamental class of Krasnov-van Hamel.

Wojciech Kucharz: Approximation and homotopy in regulous geometry.

Let X, Y be nonsingular real algebraic sets. A map $\varphi\colon X \to Y$ is said to be k-regulous, where k is a nonnegative integer, if it is of class $\mathcal{C}^k$ and the restriction of φ to some Zariski open dense subset of X is a regular map. Assuming that Y is uniformly rational, and $k \geq 1$, we prove that a $\mathcal{C}^\infty$ map $f\colon X \to Y$ can be approximated by k-regulous maps in the $\mathcal{C}^k$ topology if and only if f is homotopic to a k-regulous map. The class of uniformly rational real algebraic varieties includes spheres, Grassmannians and rational nonsingular surfaces, and is stable under blowing up nonsingular centers. Furthermore, taking $Y = \mathbb{S}^p$ (the unit p-dimensional sphere), we obtain several new results on approximation of $\mathcal{C}^\infty$ maps from X into $\mathbb{S}^p$ by k-regulous maps in the $\mathcal{C}^k$ topology, for $k \geq 0$.

Krzysztof Kurdyka : A Bochnak-Siciak theorem for Nash functions over real closed fields.

Let R be a real closed field. We prove that if R is uncountable, then a function $f\colon U \to R$ defined on an open semialgebraic set U in R^n, with $n \geq 2$, is a Nash function whenever for every affine 2-plane Q in R^n the restriction $f|_{U\cap Q}$ is a Nash function (some condition on the shape of U is required if R is not Archimedean). This is an analog of the Bochnak–Siciak theorem established in the real analytic setting. We also provide an example showing that uncountability of R is essential. Joint work with Wojciech Kucharz.

Frédéric Mangolte : Comessatti's Theorem on Rational Surfaces and Real Fano threefolds.

From the classification of real rational surfaces worked out by Comessatti at the beginning of the 20th century we get the following striking characterization of real rational surfaces: a geometrically rational real surface is rational if and only if its real locus is non-empty and connected. In a work in progress with Andrea Fanelli, we explore real loci of geometrically rational Fano threefolds and study the rationality of these.

Matilde Manzaroli : Topology of totally real degenerations.

In this talk, we study the topology of totally real semi-stable degenerations. The main result is a bound for the individual Betti numbers of a smooth real fiber in terms of the complex geometry of the degenerated fiber. The main ingredient is the use of real logarithmic geometry, which makes it possible to study degenerations that are not necessarily toric, and therefore to go beyond the case of smooth tropical degenerations, studied by Renaudineau-Shaw. This is a work in collaboration with Emiliano Ambrosi.

Adam Parusiński : On Zariski's canonical stratification of algebroid and algebraic hypersurfaces.

In this talk I discuss Zariski's construction of a canonical stratification for algebroid hypersurfaces over an algebraically closed field of characteristic zero introduced by Zariski in 1979 and extended to the algebraic hypersurfaces by Hironaka (also in 1979). This construction is based on the notion of dimensionality type that is defined recursively by considering the discriminants loci of successive "generic" corank 1 projections. The points of dimensionality type 0 are regular points and the singularities of dimensionality type 1, are generic singular points in codimension 1. Zariski proved that the latter ones are isomorphic to the equisingular families of plane curve singularities. In a joint paper with L. Paunescu, we give a similar characterization for singularities of dimensionality type 2. Moreover, we show that in this case the generic linear projections are generic in the sense of Zariski, this is still an open problem for dimensionality type greater than 2. I will also explain how we use the Eisenstein power series (in the sense of Belotto-Curmi-Rond).

Anne Pichon : Metric geometry of complex analytic germs and logarithmic link.

I will present a logarithmic version of the link of a complex analytic germ which is equipped with an ultrametric. This non-archimedean object reflects the behavior of the inner and outer metrics of the germ and it enable to describe its Lipschitz geometries through some decompositions of the logarithmic link into ultrametric balls.

Fabien Priziac : On the construction of an additive invariant for affine real algebraic varieties with action of a compact group.

I will talk about a work in progress on affine real algebraic varieties equipped with a linearizable action of a compact group. The goal of this work is to construct for these objects some additive numerical invariants with respect to equivariant homeomorphisms with algebraic graph. The study involves algebraic quotients of real algebraic sets by a polynomial action of a compact real algebraic group, free actions, arc-symmetric sets and equivariant (co)homology.

Ronan Quarez : About the central algebra of a ring.

Real algebra has been developed to answer Hilbert 17th problem and gives general postivstellensätze for polynomials non negative on a given basic closed semialgebraic subset of $\mathbb{R}^n$. For polynomials in the coordinate ring of an irreducible affine algebraic variety V a difference appears that a sum of squares in the fraction field may be nonnegative only on the central locus of V (which consists in the euclidean closure of the non-singular real closed points).

We deal with central algebra of a general domain A, studying two particular spaces of orderings : the central and the precentral loci living both in $\mathrm{Spec}_r A$, the real spectrum of A. Central orderings are of topological nature whereas precentral orderings are defined by a natural algebraic condition.

The difference between these two spaces of orderings is surprinsingly related to the stability index of A which is the infimum of the numbers k such that any basic open subset of $\mathrm{Spec}_r A$ can be described with k inequalities.

When both spaces coindide, for instance for algebraic varieties of dimension less or equal to two, we get geometric central Positivstellensätze.

This is a joint with Goulwen Fichou and Jean-Philippe Monnier.

Susanna Zimmermann : Biregulous functions, but also imperfect fields.

After talking about what I don't know about biregulous transformations of the plane, I will switch to birational transformations of the plane over imperfect fields and talk about work in progress with Fabio Bernasconi, Andrea Fanelli and Julia Schneider. Regular surfaces over imperfect fields may not be smooth or normal over the algebraic closure, which makes birational geometry over imperfect fields tricky. I want to present some other phenomena that do not appear in the birational geometry of the plane over perfect fields.