Summer school on mean field models

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Rennes, june 12-16, 2023

https ://www.lebesgue.fr/en/meanfield

Monday 12

14h-15h30 : Antoine Diez (Kyoto)

Introduction to propagation of chaos and mean-field models

The notion of propagation of chaos for large systems of interacting particles has been introduced in statistical physics in order to understand the macroscopic properties of fluids starting from a (simplified) microscopic atomic description. Over the course of the XXth century, the seminal contributions of Kac and McKean and in particular the introduction of the so-called stochastic mean-field models have paved the way to a consistent and rich mathematical kinetic theory of gas. More recently, these ideas have escaped the realm of pure statistical physics to find new applications in various areas of science where a mathematical multiscale approach is needed, for instance in biology, social sciences or data sciences to cite a few. In this talk, I will review (some of) the main and classical results induced by the works of Kac and McKean. I will then briefly discuss more recent developments of the theory in view of their new applications and try to motivate (some of) the recent and open research directions which will be discussed during the week. This presentation is based on a joint work with Louis-Pierre Chaintron.

16h-17h30 : Eva Löcherbach (Univ. Paris 1)

Mean-field limits for systems of interacting and spiking neurons, 1

I will introduce two basic processes describing systems of interacting and spiking neurons. In both processes, neurons spike at a rate depending on their membrane potential value. When spiking, they have a direct influence on their post-synaptic partners, namely, a fixed value, called “synaptic weight”, is added to the potential of the postsynaptic neurons. In between successive spikes, due to some leakage effects, the membrane potential process follows a deterministic flow. In the first class of processes, the (non-linear) Hawkes processes, the membrane potential of the spiking neuron remains unchanged upon spiking, while in the second class of process, it goes back to a resting value inducing a discontinuity that we will refer to as “big jumps”. The program is divided into 2 parts : Part I - Mean field limits of the Hawkes description of the model ; Part II - Discontinuous model with big jumps. Finally, if time allows, I will also discuss the long-time behavior of the associated non-linear limit process.

Tuesday 13

9h-10h30 : Eva Löcherbach

Mean-field limits for systems of interacting and spiking neurons, 2

11h-12h30 : François Delarue (Univ. Nice)

Mean field games and control, 1

These lectures aim to give an introduction to the theory of mean-field games and control introduced by Lasry and Lions and by Huang, Caines and Malham over fifteen years ago. The aim of these twin theories is to study equilibria in large populations of competitive or cooperative players. Mean-field games and mean-field control problems should therefore be seen as asymptotic versions of games and control problems on large populations of players with weak interaction.

In the first lecture, we will provide a heuristic derivation of mean-field games and control problems. In particular, we will define a solution of a mean-field game as a fixed point of a certain mapping and a solution of a mean-field control problem as a minimizer of a certain functional. We will show cases where the two problems are in fact related.

14h-15h30 : Pierre-Emmanuel Jabin (Penn State Univ.)

Some new developments on the mean-field limits of non-exchangeable systems, 1

Non-exchangeable multi-agent or many-particle systems encompass a wide of applications and settings : From Physics with the classical cases of point-vortices with different vorticities, point-masses in gravitational interactions to biology with integrate and fire systems of biological neurons. Those systems are characterized by the lack of symmetry between agents or particles that are not indistinguishable anymore. This may lead to significant changes in the classical theory as developed for exchangeable systems : For example, propagation of independence may not be connected anymore to the existence of a mean-field limit. This course will first present some important models that are typically non-exchangeable. We will then review some classical concepts and illustrate how their relationship may change in non-exchangeable systems. Finally the course will introduce some new recent results.

16h-16h30 : Xavier Erny (École Polytechnique)

Annealed limit and quenched control for a diffusive disordered mean-field model with random jumps

We study a sequence of N-particle mean-field systems, each driven by N simple point processes $Z^{N,i}$ in a random environment. Each $Z^{N,i}$ has the same intensity $(f(X^N_{t-}))_t$ and at every jump time of $Z^{N,i}$, the process X^N does a jump of height $U_i/\sqrt{N}$ where the U_i are disordered centered random variables attached to each particle. We prove the convergence in distribution of X^N to some limit process $\bar{X}$ that is solution to an SDE with a random environment given by a Gaussian variable, with a convergence speed for the finite-dimensional distributions. This Gaussian variable is created by a CLT as the limit of the partial sums of the U_i. To prove this result, we use a coupling for the classical CLT relying on the result of [Komlós, Major and Tusnády (1976)], that allows to compare the conditional distributions of X^N and $\bar{X}$ given the random environment, with Markovian technics.

16h30-17h15 : Lukasz Szpruch (Univ. Edinburgh)

Fisher-Rao gradient gescent for stochastic control problems

We study the convergence of Gradient and Mirror Descent schemes for approximating solutions to stochastic control problems with measure-valued controls in continuous time. By exploiting Pontryagin Optimality Principle, these rely on solving forward and backward (adjoint) equations and performing static optimisation problems regularised with Bregman divergence and can be interpreted as implicit and explicit discretisations of Fisher-Rao gradient flow. In the general (non-convex) case, we show that the objective function decreases along the gradient step. Moreover, in the (strongly) convex case, when Pontryagin Optimality Principle provides a sufficient condition for optimality, we prove that the objective converges at the (exponential) linear rate to its optimal value. The main technical difficulty is to show that stochastic control problem admits suitable relative smoothness and convexity properties. These are obtained by utilising the theory of Bounded Mean Oscillation (BMO) martingales required for estimates on the adjoint Backward Stochastic Differential Equation (BSDE).

Wednesday 14

9h-10h30 : Eva Löcherbach

Mean-field limits for systems of interacting and spiking neurons, 3

11h-12h30 : François Delarue

Mean field games and control, 2

In the second lecture, we will discuss a PDE formulation of the latter two mean-field game and control problems. In the case of control, the underlying PDE is a Hamilton-Jacobi equation defined on the space of probability measures. In games, the PDE is a more complex nonlinear equation on the space of probability measures. We will state some solvability results and give some applications to the study of the convergence problem, i.e. the passage from problems with a finite number of players to problems with an infinite number of players.

14h-15h30 : Pierre-Emmanuel Jabin

Some new developments on the mean-field limits of non-exchangeable systems, 2

16h-16h30 : William Hammersley (Univ. Nice)

Regularising gradient descents on the space of probability measures with the rearranged stochastic heat equation

This talk is about an approach to regularising by noise gradient descents in the space of probability measures that appear in mean field models. The noise is introduced in the infinite particle regime by randomising directly the non-linear stochastic differential equation of McKean-Vlasov type : first, one fixes the underlying probability space as the unit circle with Lebesgue measure, then stochastic heat is introduced to this model as the source of randomness and in order that this noise is intrinsic to this setting, we compose dynamically with the symmetric non-increasing rearrangement. This operation constrains the resulting process to be valued in a set of functions in one-to-one correspondence with the quantiles of a large class of random variables. We are able to characterise in a well-posed manner the resulting dynamics under modest conditions on the original McKean-Vlasov equation. An analysis of long time properties of corresponding stochastic gradient descents derived from potentials defined on the space of probability measures will be presented. This is joint work with François Delarue.

16h30-17h : Yi Han (Univ. Cambridge)

Stochastic PDEs on Hilbert space with irregular noise coefficients

In the first part I review how ideas based on relative entropy could help us solve some McKean-Vlasov SDEs and SPDEs with irregular interaction term. I will also review the smoothing properties of McKean-Vlasov equations with an interaction kernel. In the second part I discuss how a generalized coupling technique can be applied to find unique solution to Hilbert space valued SPDEs with merely Hölder continuous diffusion coefficient, covering both the stochastic heat and wave equations. The diffusion coefficient is non-degenerate, and the assumed Hölder regularity threshold depends on the eigenvalues of the operator A in a quantitative way. Long time behaviors of the solution and small mass limit of the damped wave equation are also dealt with via this framework.

17h-17h30 : Katharina Schuh (TU Wien)

Global contractivity for Langevin dynamics with distribution-dependent forces and uniform in time propagation of chaos

We study the long-time behaviour of both the classical second-order Langevin dynamics and the nonlinear second-order Langevin dynamics of McKean-Vlasov type. We establish global contraction in an L^1-Wasserstein distance with an explicit dimension-free rate for pairwise weak interactions. The contraction result is not restricted to external forces corresponding to strongly convex confining potentials. It rather includes multi-well potentials and non-gradient-type external forces as well as non-gradient-type repulsive and attractive interaction forces. In the proof, we use a coupling approach and construct a distance function adjusted to it. By applying a componentwise adaptation of the coupling we obtain uniform in time propagation of chaos bounds for the corresponding mean-field particle system. The talk is based on arXiv :2206.03082.

Thursday 15

9h-10h30 : François Delarue

Mean field games and control, 3

In the third course, we will look at other issues when the problems in question (game or control) can have multiple solutions. In this situation, the first two PDEs may not have classical solutions. Some works in progress address the notions of solution that should be used in such cases. Another question concerns the study of the convergence problem when uniqueness is lost in the asymptotic formulation.

11h-12h30 : Pierre-Emmanuel Jabin

Some new developments on the mean-field limits of non-exchangeable systems, 3

14h-14h30 : Thomas Cavallazzi (Univ. Rennes)

Quantitative weak propagation of chaos for McKean-Vlasov SDEs driven by α-stable processes

In this talk, we will deal with McKean-Vlasov SDEs driven by α-stable processes, with $\alpha \in (1, 2)$. We make Hölder-type assumptions on the coefficients, with respect to both space and measure variables. We will study the associated semigroup, acting on functions defined on the space of probability measures, through the related backward Kolmogorov PDE describing its dynamics. We will focus in particular on its regularizing properties. We will finally use the preceding results to prove quantitative weak propagation of chaos for the associated mean-field interacting particle system.

14h30-15h : Léo Hahn (Univ. CLermont Auvergne)

Invariant measure and mixing behavior of a pair of run-and-tumble processes with hard-core interactions

Run-and-tumble particles are a paradigmatic model in out-of-equilibrium physics that exhibits interesting phenomena not found in their passive counterparts such as motility-induced phase separation. I will present the long-time behavior of a pair of such particles with hard-core interactions on a unidimensional torus by casting them as a piecewise deterministic Markov process. I will show that this continuous-space process is the distributional limit of a known on-lattice model for particles exhibiting persistent motion and jamming interactions. Finally, I will discuss the scaling of the continuous-space mixing time as a function of model parameters, which exhibits a persistent and a diffusive regime

15h-15h30 : Pierre Le Bris (Sorbonne Univ.)

An observation concerning the effect of the Random Batch Method on phase transition

In this talk, we focus on the numerical simulation of a system of N particles in mean-field interaction. To deal with the quadratic complexity of the numerical scheme, corresponding to the computation of the $O(N^2)$ interactions per time step, the Random Batch Method (RBM) has been suggested. It consists in randomly (and uniformly) dividing the particles into batches of fixed size $p > 1$, and computing the interactions only within each batch, thus reducing the numerical complexity to $O(Np)$ per time step. The convergence of this numerical method has been proved in other works. This talk is motivated by the observation that the RBM, via the random constructions of batches, artificially adds noise to the particle system. The goal is to study the effect of this added noise on the possible phase transition of the nonlinear limit. To study this phenomenon, we focus on two toy models : the Curie-Weiss model and the system of N particles in linear interactions in a double well confining potential. Both models, which have been extensively studied, describe a large system of particles with a mean-field limit that admits a phase transition. We study the effective dynamics of the models to show how the critical temperature decreases with the RBM.

16h-16h30 : Marta Leocata (Scuola Normale Superiore, Pisa)

Some variations on the mean-field limit : different types of interactions and the first-order approximation

The purpose of this talk is twofold, first of all, I will present two variations on the mean-field limit comparing the results with the classical theory and then I will share ideas on an on-going project of an approximation result using the mean-field limit and its fluctuations. To begin, a brief overview of the comparison between mean-field, intermediate and local interactions will be given. Then the first result will show how the intermediate interaction can be fruitful in some application contexts related to tissue formation. The second result concerns the derivation of the PDE limit when the local interaction is considered (the nature of our results is mainly heuristic and numerical). To conclude, I will consider the case of particles interacting according to a mean-field interaction. We propose to study an approximation for the particle system given by an SPDE. This SPDE differs from the purely deterministic one given by the McKean-Vlasov PDE, because fluctuations are also represented in it. This equation is known in the literature as a first-order approximation.

16h30-17h15 : Michela Ottobre (Heriot-Watt Univ.)

McKean-Vlasov S(P)Des with additive noise

Many systems in the applied sciences are made of a large number of particles. One is often not interested in the detailed behaviour of each particle but rather in the collective behaviour of the group. An established methodology in statistical mechanics and kinetic theory allows one to study the limit as the number of particles N tends to infinity and to obtain a (low dimensional) PDE for the evolution of the density of particles. The limiting PDE is a non-linear equation, where the non-linearity has a specific structure and is called a McKean-Vlasov nonlinearity. Even if the particles evolve according to a stochastic differential equation, the limiting equation is deterministic, as long as the particles are subject to independent sources of noise. If the particles are subject to the same noise (common noise) then the limit is given by a Stochastic Partial Differential Equation (SPDE). In the latter case the limiting SPDE is substantially the McKean-Vlasov PDE + noise ; noise is further more multiplicative and has gradient structure. One may then ask the question about whether it is possible to obtain McKean-Vlasov SPDEs with additive noise from particle systems. We will explain how to address this question, by studying limits of weighted particle systems, in a framework introduced by Kurtz and collaborators.

This is a joint work with L. Angeli, D. Crisan, M. Kolodziejzik.

Friday 16

9h15-10h : Eric Luçon (Univ. Paris Cité)

How large is the mean-field framework ? LLN and CLT for empirical measures of diffusions on (random) graphs.

A common hypothesis for mean-field models is to suppose all-to-all interaction between N particles (the interaction is on the complete graph) with uniform coupling along the population. The motivation of the talk is simple : what happens if one removes edges in the graph of interaction ? the coupling is no longer a functional of the empirical measure of the system, but rather of local empirical measures around each vertex. Under what condition on the graph of interaction do these empirical measures behave as in the mean-field framework as $N \to \infty$? We address this question of universality both at the level of the law of large numbers and fluctuations, for a large class of possibly random graphs, including the Erdös-Rényi class. This is based on joint works with S. Delattre, G. Giacomin and F. Coppini and C. Poquet.

10h-10h30 : Rémi Moreau (Univ. Rennes)

Constrained in law BSDE and associated particle system

In this talk, we first consider a BSDE constrained to a common noise. We study the associated particle system and prove propagation of chaos. Then, relying on results from a previous article by Briand et al., we add a normal constraint in conditional law and adapt the results from the aforementioned work.

11h-11h30 : Ke Yan (Univ. Rennes)

Extended mean-field control problem with partial observation

In this talk, we study an extended mean-field control problem with partial observation, in which the dynamic of the state is given by a forward-backward stochastic differential equation of McKean-Vlasov type. The cost functional, the state and the observation all depend on the joint distribution of the state and the control process. We first establish a necessary condition in the form of Pontryagin's maximum principle for optimality under the reference probability space. Then a verification theorem is obtained for optimal control under some convex conditions of the Hamiltonian function. This talk is based on a joint work with Prof. Tianyang Nie.

11h30-12h : Yoan Tardy (Sorbonne Univ.)

Convergence of the empirical measure for the Keller-Segel model in both subcritical and critical cases.

We show the weak convergence, up to extraction of a subsequence, of the empirical measure for the Keller-Segel system of particles in both subcritical and critical cases. We use a simple two particles moment argument in order to show that the particle system does not explode in finite time, uniformly in the number of particles in some sense.

12h-12h45 : Milica Tomasevic (CNRS - École Polytechnique)

Particle approximation of the doubly parabolic Keller-Segel equation in the plane

In this work, we study a stochastic system of N particles associated with the parabolic-parabolic Keller-Segel system in the plane. This particle system is singular and non Markovian in that its drift term depends on the past of the particles. When the sensitivity parameter is sufficiently small, we show that this particle system indeed exists for any $N \geq 2$, we show tightness in N of its empirical measure, and that any weak limit point of this empirical measure, as $N \to \infty$, solves some nonlinear martingale problem, which in particular implies that its family of time-marginals solves the parabolic-parabolic Keller-Segel system in some weak sense. The main argument of the proof consists of a "Markovianization" of the interaction kernel : We show that, in some loose sense, the two-by-two path-dependant interaction can be controlled by a two-by-two Coulomb interaction, as in the parabolic-elliptic case. This is a joint work with N. Fournier (LPSM, Sorbonne Univ.).