Lagrangian trajectory reconstruction using ensemble-based data assimilation technique: a first approach

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Particle tracking velocimetry (PTV), also referred to as Lagrangian particle tracking (LPT) has recently gained considerable revival. The trend started with the Iterative Particle Reconstruction (IPR) method [1] that applied a projection-matching scheme, to reconstruct 3D particles' positions rather than voxel-based intensity, like in Tomographic PIV [2]. Recently, IPR has given rise to the Shake-The-Box (STB) method [3] able to tackle densely seeded flows with considerably high accuracies and reasonable computational efforts. The success of STB hinged on two factors: the image matching schemes as in IPR and a dynamical predictor exploiting the temporal coherence of the particle image data respectively. However, in most of 3D turbulent flows, image-based experiments can only provide sparse spatiotemporal data, for which STB is not able to track particles. Yet, dynamical models are more predictive than Wiener filter or polynomial extrapolations. If more robust estimations are possible, something useful may be learnt from the coupling between dynamical models and image data.

In responding to these problems, we introduce a novel approach originated from the data assimilation technique comprising a sampling-based optimal estimation algorithm, namely a group of ensemble-based filtering variational schemes [4]. Similar to STB, the reconstruction process is divided into two phases: in the prediction phase, at time level k, the predictor $\mathcal{F}$ gives the particle's position based on prior information:

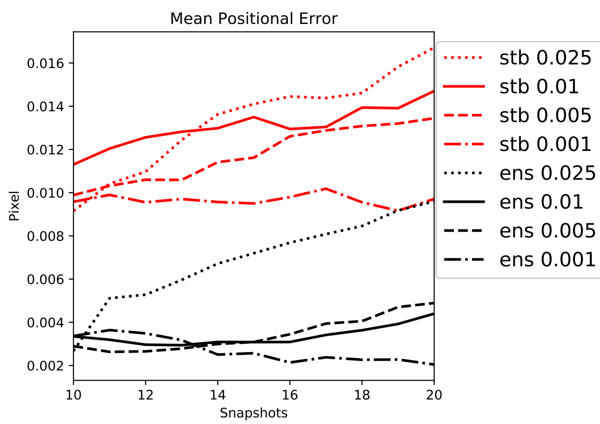
$$\theta_{p,k} = \mathcal{F}_k(\theta_{p,k-1}, ..., \theta_{p,0}), \tag{1}$$

where the state vector θ denotes the coordinates and the intensity of one particle p. In the correction phase the predicted position is corrected by assimilating the corresponding image data. Thus, we search for $\delta\theta_p$ the variation of state of particle p that minimize the following cost function:

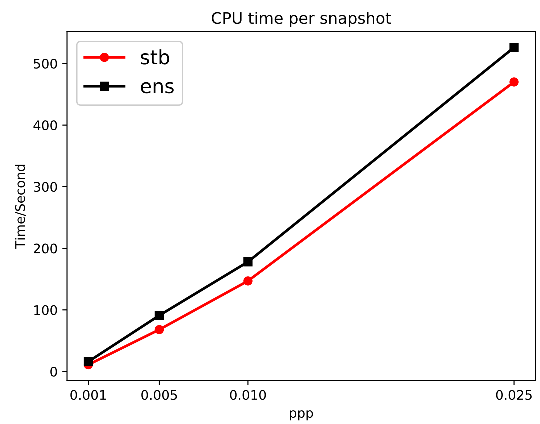
$$J(\delta\theta_p) = \sum_i ||D_p^i - \partial_\theta I_{part}^i \delta\theta_p||^2, \tag{2}$$

where I_{part}^i is the observation operator of camera i linking the 3D particle's position in object space and its 2D image projection in image space incorporating the image matching scheme used in IPR/STB. The symbol ∂_θ denotes a linearization operator about current particle position and D_p corresponds to the residual image. The corrected positional field improves the accuracy of the predictor and gradually leads to reconstruction field of better quality. We find it especially beneficial to employ an ensemble-based optimal estimation method partly because the optimization is done, analogue to STB, by linearizing the original non-linear optimization problem to a linear quadratic form (2). More importantly, we have shown that the linearization through the ensemble, comprising a cloud of possible positional state of all particles, is able to explore a larger search radius than the quadratic points fitting algorithm used in STB. Furthermore, the ensemble formulation naturally incorporates the uncertainties of the state that is consistent with the dynamics. Nevertheless in the present study, we use the same polynomial-based predictor in ensemble-based method as in STB. The goal of this work is not to promote the ensemble-based method as an alternative to STB method, but rather to introduce the ensemble approach as an optimal estimation framework. This framework allows us to couple the particle image based data, as well as the prior information brought in by sophisticated CFD models through a robust stochastic searching algorithm combined with the effective image matching schemes.

The proposed method is quantitatively evaluated with synthetic particle image data. Firstly the synthetic tracks are created by transporting virtual particles in a turbulent cylinder wake-flow at Reynolds number equal to 3900. The Eulerian velocity data was obtained with a Large Eddy Simulation (LES)[5]. The first few snapshots of particles' tracks constitute the prior distribution of particles for prediction phase of both ensemble-based method and STB. The synthetic tracks are added by Gaussian errors and then projected on 4 virtual cameras in order to produce the particle image time series data. Both the image data and the prior particle distribution of several snapshots are passed to the proposed approach along with a naively-implemented STB approach for 3D particle tracks' reconstruction. A simplified triangulation process is applied to retrieve the positions of new particles entering the domain. Finally we examined the mean positional error of the reconstructed particles, the required CPU time and memory as well as the occurrence of ghost particles. Figure 1a compares the temporal evolution of the mean positional error over all particles of all cameras of the two approaches. Figure 1b shows the comparison of sequential CPU run time spent by different method with respect to different ppp levels (note that the current STB and ensemble methods' implementation can only support sequential run). The mean positional error of ensemble method is more accurate than that of the STB. This is partly because the STB approach used in the evaluation is naively implemented therefore requires further fine tuning of the parameters. On the other hand, the ensemble method performs better with moderately elevated computational time. The positional error could be further reduced to the level of $1e^{-4}$ px if more samples are employed at the cost of higher computational power. These preliminary results indicated that the ensemble-based method is indeed effective. Current investigations include dealing with image of higher ppp level and incorporating sophisticated dynamic model. The results will be reported in the following paper.



(a)



(b)

Figure 1: *(a):The temporal evolution of the mean positional error along time series image data of STB/ENS under different ppp level. (b): The sequential CPU time spent by STB/ENS with respect to different ppp levels.*

References

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